GILL'S SCHOOL SERIES

SCHOOL OF ART GEOMETRY

BY

GEORGE GILL,

WITH AN APPENDIX ON EXERCISES IN SOLID GEOMETRY

BY A. J. BRIANT.

LIVERPOOL.
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SCHOOL OF ART GEOMETRY.

Gill's School Series.

THE SCHOOL OF ART

GEOMETRY.

BY

GEORGE GILL,

Author of "First Grade Practical Geometry," "Second Grade Plane and Solid Geometry," "First, Second, Third and Fourth Oxford Readers," "Oxford and Cambridge Geography," &c., &c.

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PREFACE.

THIS text book on Practical Plane and Solid Geometry is especially intended for the use of students in Schools of Art who are preparing for the *Second Grade* Drawing Examination held by the Science and Art Department; therefore it contains the whole of the First and Second Grade Courses as taken in Elementary Schools, as well as various additions suitable for pupils of more mature years.

Since the issue of my old Second Grade Geometry, which has been so generally used in the Schools of Art and Grammar Schools of the country, many modifications have been made in the scope of work required from the candidates. Much more attention has now to be given to *proportional and equivalent figures—to the use of scales—to tangents to two circles—to foiled figures*, and other problems which are so extensively applied in the construction of *mechanical patterns* and *geometrical tracery* in architecture—hence the chapters bearing on these especial branches of study have been much enlarged, whilst a suitable course of easy lessons on *Projection* has been added to suit the requirements of the Department in Solid Geometry.

Although I have anxiously endeavoured not to encumber the work by unnecessary expansion, still I have carefully given every problem in Plane and Solid Geometry which contains a principle of construction the Second Grade pupil is expected to know and apply.

GEORGE GILL.

NEW BRIGHTON, CHESHIRE,
September 1st, 1874.

Contents.

PLANE GEOMETRY.

SOLID GEOMETRY.

School of Art Practical Geometry.

SECTION I.

DEFINITIONS.

A **Point** is used for marking position only ; it has no size.

Although for the sake of convenience we make a black dot as A to indicate a point, still the true point is not the whole of the dot, which has size, but merely the centre of the dot.

LINES.

A **Line** has length only, and no breadth or thickness. Lines are either straight or curved.

The ends of lines are points, as A and B.

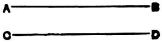
A **Straight Line** is the shortest distance between two points. **Curved Lines** are no where straight.

A *straight line* is sometimes called a *right line.*

There are *three kinds* of straight lines—**Horizontal, Perpendicular, and Oblique.**

Horizontal. Perpendicular. Oblique.

Parallel Lines are the same distance from each other throughout their entire length.



Straight lines, which are Parallel, never meet if drawn ever so far any way. A B is parallel to C D.

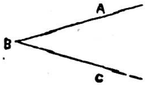
Surfaces have *length* and *breadth* only but no thickness. Surfaces are bounded by lines.

The spaces enclosed by the lines in the adjoining figures are called surfaces. A surface is sometimes called a *superficies*. Any surface perfectly flat everywhere is called a *plane surface*.

ANGLES.

An Angle is the opening between two straight lines which meet at a point.

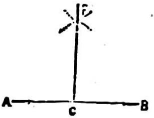
The two lines, B A and B C, make the angle called A B C. They meet at the point B and therefore the angle is sometimes named the angle B.



There are *three kinds* of angles—the **right** angle, the **obtuse** angle, and the **acute** angle.

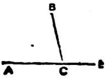
Right Angles.—When one straight line falls on another straight line, and makes the angles on each side of it equal to each other, each of the angles is a *right angle*, and the line making these angles is called a *perpendicular*.

In this illustration, the line F C falls on the line A B, and makes the angle A C F equal to the angle B C F, therefore each of the angles, A C F and B C F, is a right angle, and the straight line F C is perpendicular to A B.



Obtuse Angle.—An *obtuse angle* is greater than a right angle. The angle B C E is an obtuse angle.

Acute Angle.—An *acute angle* is less than a right angle. The angle A C B is an acute angle.



TRIANGLES.

A Triangle is a figure enclosed by three straight lines. Triangle have *three sides* and *three angles.*

There are *three kinds of triangles* named after their sides, viz.:—
Equilateral, Isosceles, and Scalene.

Fig. 1.

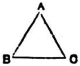


Fig. 2.

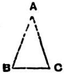
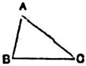


Fig. 3.



An Equilateral Triangle has its three sides equal, as Fig. 1.

An Isosceles Triangle has two sides equal, as Fig. 2.

A Scalene Triangle has none of its sides equal as Fig. 3.

There are *three kinds of triangles* named after their angles, viz.:—a **Right-angled Triangle**, an **Obtuse-angled Triangle**, and an **Acute-angled Triangle.**

Fig. 1.

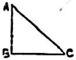
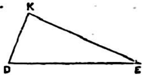


Fig. 2.

Fig. 3.



A Right-angled Triangle has one right angle, as the angle B in the triangle A B C, Fig. 1.

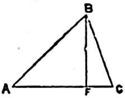
An Obtuse-angled Triangle has one of its angles an obtuse angle, as the angle B in the triangle A B C, Fig. 2.

An Acute-angled Triangle has all its angles acute, as the angles in the triangle D K E, Fig. 3.

PARTS OF A TRIANGLE.

The **base** of a triangle is its lowest side, as A C in the triangle A B C.

The **Vertex** of a triangle is the angle opposite the base, as B in the triangle A B C.



The **Perpendicular Height** of a triangle is measured by a perpendicular line let fall from the vertex upon the base, as the line B F in the triangle A B C. (See the last illustration.)

QUADRILATERAL OR FOUR-SIDED FIGURES.

A figure with four sides is called a **Quadrilateral Figure** or a **Quadrangle.**

A **Parallelogram** is a quadrilateral figure, having its opposite sides equal and parallel.

Parallelograms are of four kinds—the **Square, Rectangle** or **Oblong, Rhombus,** and **Rhomboid.**

Fig. 1.

Fig. 2.

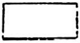


Fig. 3.

Fig. 4.

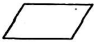
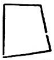


Fig. 5.



A **Square** is a parallelogram whose four sides are equal, and whose angles are all right angles, as Fig. 1.

A **Rectangle** or **Oblong** is a parallelogram whose opposite sides only are equal, but whose angles are all right angles, as Fig. 2.

A **Rhombus** is a parallelogram whose four sides are equal, but whose angles are not right angles, as Fig. 3.

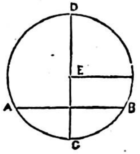
A **Rhomboid** is a parallelogram whose opposite sides only are equal, and whose angles are not right angles, as Fig. 4.

A **Trapezium** is a quadrilateral figure, none of whose sides are parallel, as Fig. 5.

A four-sided figure, which has only two of its sides parallel, is called a Trapezoid.

The line that joins any two of the opposite angles of a quadrilateral figure is called its **Diagonal.**

THE CIRCLE.



A **Circle** is a figure bounded by a curved line, called its **Circumference**, which is everywhere the same distance from a point called its **Centre**. E is the centre of the above circle.

A **Radius** is a straight line drawn from the *centre* to the *circumference* of a circle, as E D or E F.—Plural form "*radii.*"

A **Diameter** is a straight line drawn through the centre of a circle from any two opposite points in the circumference, as C D.

An **Arc** of a circle is any portion of its *circumference*, as A C, or D B.

A **Chord** of a circle is a straight line which joins the ends of an arc,

A B is a chord both of the arc A C B and A D B.

A **Segment** of a circle is any portion of it cut off by a *chord.*

The figure enclosed by the arc A C B, and the chord A B, is a segment of the circle.

A **Sector** of a circle is any portion of it contained within two of its radii and an arc, as F E C.

A **Semi-circle** is half a circle, as D B C or D A C.

A **Quadrant** is the fourth part of a circle, as F E C.

Every circle is supposed to be divided into 360 equal parts, called *degrees*; therefore, the arc of every *semicircle* contains 180°, and of every quadrant 90°. All angles are measured by means of arcs. If we fix a compass on the angular point, and describe an arc between the two arms of an angle, this angle is said to contain as many degrees as there are in the arc. Thus the angle of a quadrant, which is a right angle, contains 90°. If the arc of an angle contains 30°, the angle itself is said to be an angle of 30°. For measuring angles, see the Protractor, page 3.

POLYGONS.

Figures that have more than four sides are called **Polygons.**

A Pentagon	has	five	sides.
A Hexagon	„	six	„
A Heptagon	„	seven	„
An Octagon	„	eight	„
A Nonagon	„	nine	„
A Decagon	„	ten	„
An Undecagon	„	eleven	„
A Duodecagon	„	twelve	„

There are two kinds of Polygons—*Regular* and *Irregular.*

A **Regular** polygon has all its sides and angles equal; an **Irregular** polygon has its sides and angles unequal.

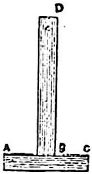
Similar figures are *equi-angular,* and their corresponding sides are *proportional.*

DRAWING INSTRUMENTS.

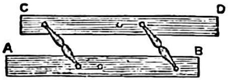
In addition to *Lead Pencils, Dividers, Pencil Compasses,* a plain *Scale of Inches* divided into eighths, and *Two Set Squares,* which are absolutely required, the pupil should also have a T *Square, Parallel Ruler,* and *Protractor.*

THE T SQUARE.

The T *Square* consists of two straight rulers fixed at right angles to each other, as shown in this illustration, and is chiefly used for drawing perpendicular lines. If the edge A C be placed along a straight line, the edge D B will give the direction of a line at right angles to it.

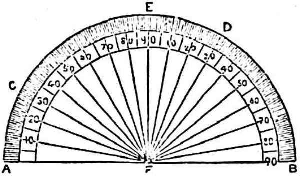


THE PARALLEL RULER.



The *Parallel Ruler* is used for drawing parallel lines, and consists of two rulers fixed parallel to each other by means of two equal brass links which are fastened to the rulers at equal distances by pivots. If the edge A B be placed along a straight line, the edge C D will give the direction of a line parallel to it.

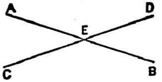
THE PROTRACTOR.



The *Protractor* is used for measuring angles. It usually consists of a thin semicircle of brass, which is divided into degrees as shown in this illustration. A line can be drawn at any angle from a point in a given line by placing the edge A B along the line, so that the point F rests on the given point. For example, the line F C on the Protractor makes with the line A F an angle of 30°, and F D makes A F with an angle of 120°, and with F B an angle of 60°.

USEFUL PROPOSITIONS.

1st.—When a straight line falls on another straight line, it makes the adjacent angles equal together to two right angles.



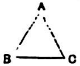
Explanation.—The angles A E D and D E B, made by the straight line D E falling on A B, are equal together to two right angles.

2nd.—If two straight lines cross one another the opposite angles are equal.

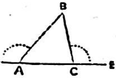
Explanation.—The opposite angles A E C and D E B are equal to each other. and likewise the opposite angles A E D and C E B.

The four angles made by any two straight lines crossing one another are equal together to four right angles; for instance, the four angles A E C, C E B, D E B and A E D are equal together to four right angles.

3rd.—The three angles of any triangle are together equal to two right angles.

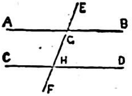


Explanation.—The three angles A B C, B C A, and C A B are equal together to two right angles.



Explanation.—The exterior angle B C E is equal to the two interior angles B A C and A B C.

4th.—If a straight line crosses two parallel straight lines it makes the alternate angles equal.



Explanation.—The angle A G H is equal to the alternate angle G H D; and also the angle B G H to the alternate angle G H C.

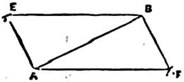
5th.—If a straight line crosses two parallel straight lines it makes the exterior angle equal to the interior and opposite angle.

Explanation.—The exterior angle E G B is equal to the interior and opposite angle G H D; and likewise the angle E G A is equal to the angle G H C.

Also the two angles B G H and G H D are equal together to two right angles.

6th.—The opposite sides and angles of a Parallelogram are equal to each other, and the diameter divides it into two equal parts.

Explanation.—The sides A E and F B are equal, and also the sides E B and A F.
The opposite angles A E B and A F B are equal, and likewise the angles E A F and E B F.
The diameter A B bisects the Parallelogram A E B F, that is, the triangle A E B is equal to the triangle A F B.



USEFUL PRELIMINARY PROBLEMS.

Such Exercises as the following should be given at this stage :—

1. Draw lines measuring respectively $5''$,* $4\frac{3}{8}''$, $2\frac{7}{8}''$, $1\frac{1}{8}''$, and any other given lengths.

2. Draw any line A B, $3''$ long, and step off on it from A five equal distances. (Take A B, horizontal, perpendicular, and oblique.)

3. Give points and request the pupils to connect them by means of various kinds of lines. (Note.—Adjust the ruler to the pencil, and not the pencil to the ruler.

4. Draw lines in various directions, and request the pupils to draw others parallel to them, say an inch or any other distance from them, by means of the set squares.

5. Draw lines in various directions, and request the pupils to erect perpendiculars at certain points by means of the set squares.

* Note.—Feet are often expressed by one dash; thus $5'$ means 5 feet, and inches by two dashes; thus $5''$ means 5 inches.

SECTION II.

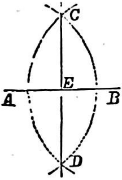
The Problems contained in Sections 1, 2, 3 and 4, belong to the First Grade course; Second Grade pupils, however, must know them.

LINES.

The pupil must practice these problems with the lines in various positions.

PROBLEM I.

To bisect a given straight *or* curved line A B.

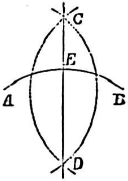


1st.—From A as centre, *with any radius greater than half the line*, describe an arc.

2nd.—From B as centre, *and with the same radius*, cut this arc in C and D.

3rd.—Join C D by a line cutting A B in E, then

A B is bisected in E.



PROBLEMS II. & III.

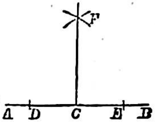
To draw a straight line perpendicular *to a given straight line* A B, *from a given point* C, *in the line.*

Case 1.

Prob. 2.—*When the point is* AT *or* NEAR THE MIDDLE *of the line.*

Let C be the point. From C, *with any radius*, cut A B in D and E. From D and E, *with any radius*, describe arcs intersecting in F. Join F C, then

*Note.—F C is the required perpendicular.



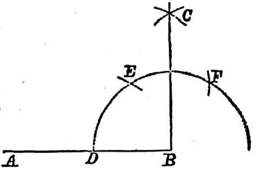
* Also required from Second Grade pupils.

Case 2.

Prob. 3.—*When the point is* AT *or* NEAR ONE END *of the line* A B.

Let B be the point. From B, *with any radius*, describe arc D E F. From D, *with same radius*, step off the distances E and F. From E and F, *with any radius*, describe arcs intersecting in C. Join B C, then

B C is the required perpendicular.



PROBLEMS IV. & V.

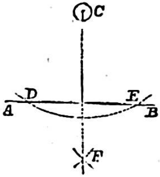
To draw a straight line **perpendicular** *to a given straight line* A B, *from a given point outside it.*

Case 1.

Prob. 4.—*When the point is* OPPOSITE *or* NEARLY OPPOSITE *the* MIDDLE *of the line* A B.

Let C be the point. From C, *with sufficient radius*, cut A B in D and E. From D and E as centres, *with any radius*, describe arcs intersecting in F. Join F C, then the line

F C is the required perpendicular.

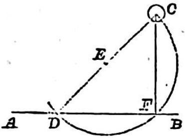


Case 2.

Prob. 5.—*When the point is* OPPOSITE *or* NEARLY OPPOSITE *the extremity of* A B.

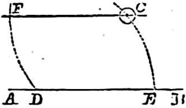
Let C be the point. Take any point D, in A B, not opposite C. Join D C and bisect it in E (P 1.) From E as centre, *with* E D *as radius*, describe an arc cutting A B in F. Join C F, then the line

C F is the required perpendicular.



PROBLEM VI.

To draw a line parallel *to a given line* A B, *through a given point* C.

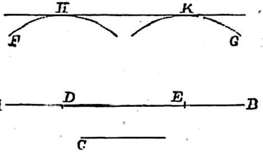


In A B take any point, D, *not opposite* C. From D as centre, with D C *as radius*, describe an arc C E; and from C as centre, *with the same radius*, describe the arc D F. Make D F equal to E C. Draw the line F C, then

F C will be parallel to A B.

PROBLEM VII.

To draw a line parallel *to a given line* A B, *at a given distance from it, equal to* C.



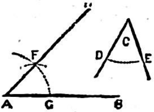
Take any two points, D and E in the line A B as centres From these, *with* C *as radius*, describe arcs F and G Draw the line H K touching these arcs, then

H K is parallel to A B at a distance from it equal to C

ON ANGLES.

PROBLEM VIII.

From point A, *draw a line, making with the given line* A B, *an* angle equal *to the given angle* C.

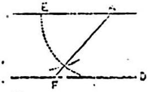


From point C, *with any radius*, describe the arc D E. From point A *with same radius*, describe arc F G. Make the arc F G equal to the arc D E. Through F, draw line A F H, then

H A B is the required angle, equal to angle C.

PROBLEM IX.

To draw a line from a given point A, *outside a given line* B D, *making with the given line, an* angle equal *to a given angle* C.



Let C, *Prob.* 8, *be the given angle.*

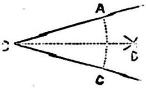
Through the point A, draw a line A E, parallel to the given line B D (P. 6.) Draw A F, making with the line E A an angle equal to C (P. 8,) and cutting B D in F, then

A F D is the angle required, equal to angle C., Problem VIII.

Note.—Problems 6, 8, and 9, in particular, should be frequently worked with the line and point in various positions.

PROBLEM X.

To **bisect a given angle A B C.**

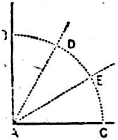


From B, *with any radius,* describe an arc cutting the arms of the angle in A and C. From A and C, *with any radius,* describe arcs cutting in D. Draw line B D, then

B D bisects the angle A B C.

PROBLEM XI.

To **trisect a right angle A.**



From A, *with any radius,* describe an arc, cutting the arms of the angle in B and C. From B and C, as centres, *with the same radius,* cut the arc in E and D. Draw the lines A D and A E. Then the lines

A D and A E trisect the right angle B A C.

N.B.—A right angle contains 90°, therefore B A C is an angle of 90°; E A C an angle of 30°, and D A C an angle of 60° An angle of 45° is half a right angle; an angle of 15° is half an angle of 30°. Hence the pupil will find no difficulty in constructing, without a protractor, angles of 90°, 75°, 60°, 45°, 30°, and 15°, which the teacher should require him to do.

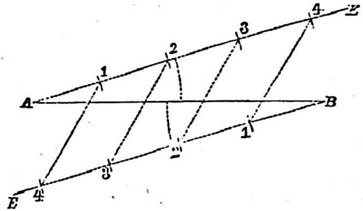
Note.—*The pupil should frequently be required to make angles of various degrees and in any positions, by means of the compass and ruler.*

ON DIVIDED LINES.

PROBLEMS XII & XIII.

To **divide a straight line A B** *into any number of equal parts. For example take 5.*

FIRST METHOD.



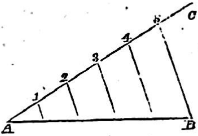
Draw the line A E, at any angle to A B (P. 3.) Through B, draw the line B E, parallel to A E (P. 6.) With any convenient radius, step off from A and B on the lines A E and B E the number of parts, less one, into which the line A B is required to be divided, as 1, 2, 3, 4. Join 1 and 4, 2 and 3, 3 and 2, 4 and 1; then

A B is divided into five equal parts.

SECOND METHOD.

At A draw any line A C of unlimited length, making any angle with A B. Mark off any five equal distances from A towards C. Join 5 and B. Through the remaining divisions 1, 2, 3, and 4, draw lines falling on A B and parallel to 5 B, then

A B is divided into five equal parts.



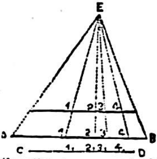
Note.—*The pupil should now divide a line into 7, 10, or any other number of equal parts.*

PROBLEM XIV.

To **divide a given line A B proportionally** *to a given divided line* C D.

Draw a line parallel to A B equal to C D, and similarly divided, *at any distance from it.* Join A and B with the ends of this line, and produce them to meet in E. Draw lines from E, through the divisions 1, 2, 3, 4, meeting A B in 1, 2, 3, 4, then

A B is divided proportionally to C D.



Note.—The same principle divides a shorter line than the given one, only the apex of the triangle would fall below A B instead of above.

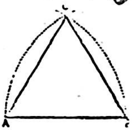
ON TRIANGLES.

PROBLEM XV.

To construct an **equilateral triangle** *on a given base* A B.

From A and B as centres, *with A B as radius*, describe arcs cutting each other in C. Join A C and B C, then

A B C is the equilateral triangle required.

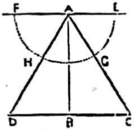


PROBLEM XVI.

To construct an **equilateral triangle,** *having a given height* A B.

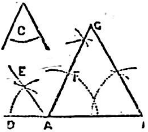
From the extremities of the line A B, draw the lines F A E, and D B C perpendicular to it. (P. 3.) From A as centre, *with any radius*, describe a semicircle cutting F A E in F and E. From E and F, with *same radius*, cut this semicircle in G and H. Draw lines from A through G and H, meeting C D in C and D, then

A D C is the equilateral triangle required.



PROBLEM XVII.

To construct an isosceles triangle, *having its base* A B, *and opposite angle* C, *given.*



Produce the base to D, and from A draw a line A E, making with A D an angle equal to the given angle C (P. 8.) Bisect the angle E A B by the line A F (P. 10.) From B, draw a line, making with B A, an angle equal to the angle F A B (P. 8.), and meeting A F produced in G, then

A G B is the isosceles triangle required.*

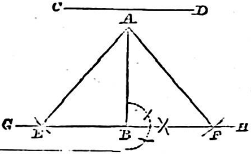
Note.—The pupil should be taught the reason for this method of construction, otherwise he will be apt to forget it.

PROBLEM XVIII

Construct an isosceles triangle, *of which* A B *is the perpendicular height, and* C D *the length of each of the equal sides.*

Through B draw a line G H of indefinite length at right angles to A B. (Prob. 3.) From centre A, with distance C D, mark off on G H the points E and F. Join A E and A F. Then

A E F is the isosceles triangle required.

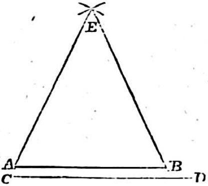


PROBLEM XIX.

Construct an isosceles triangle, *of which* A B *is the base, and* C D *the length of each of the equal sides.*

From centres A and B, with C D as radius, describe arcs intersecting in E. Join A E and B E. Then

the triangle A E B is the isosceles triangle required.

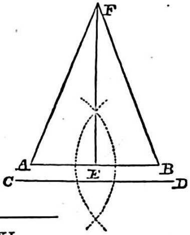


* N.B.—Isosceles triangles having angles at the apex of 30°, 45°, 60°, 75°, and 15° respectively should now be constructed.

PROBLEM XX.

Construct an isosceles triangle, *of which* A B *is the base, and* C D *the perpendicular height.*

Note.—The apex of an isosceles triangle stands exactly over the centre of the base.

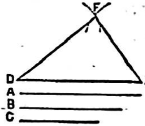


1. Bisect the base A B in E.
2. At E erect a perpendicular E F equal to C D. (Prob. 2.) Join A F and B F. Then

A B F is the isosceles triangle required.

PROBLEM XXI.

To construct a triangle, *having its sides equal to three given lines,* A, B, *and* C.

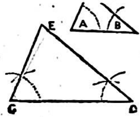


Draw a line D E equal to A. From D as centre, *with* B *as radius,* and from E as centre, *with* C *as radius,* describe arcs cutting in F. Join D F and E F, then

D F E is the triangle required.

PROBLEM XXII.

To construct any triangle, *its base* C D *being given, and having angles at the base equal to given angles.*



Let A and B be the given angles.

At C and D make angles E C D and E D C, equal to the given angles A and B, by lines meeting in E, (Prob. 8), then

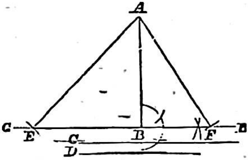
E C D is the triangle required.

PROBLEM XXIII.

Construct a scalene triangle, *of which* A B *is the perpendicular height, and* C *and* D *the length of the sides meeting in the apex* A.

Through B draw a straight line G H of indefinite length at right angles to A B. (Prob. 3.) From A, with distances C and D respectively, mark off on G H the points E and F. Join A E and A F. Then

A E F is the scalene triangle required.



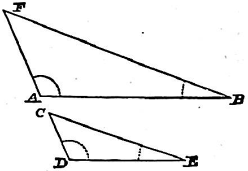
PROBLEM XXIV.

On base A B *construct a* triangle *similar to the given triangle* C D E.

N.B.—Triangles are similar to each other when their angles are equal.

Make angles at A and B equal respectively to the angles at D and E, producing the lines to meet in F. Then the angle A F B is also equal to the angle D C E, and

the triangle A F B is the similar triangle required.

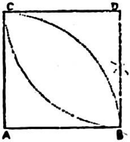


Note.—*The corresponding sides of similar triangles are proportional, that is, the three sides of the triangle* A F B *are proportional to the corresponding sides of the triangle* D C E.

QUADRILATERAL FIGURES.

PROBLEM XXV.

To construct a square *upon a given base* A B.

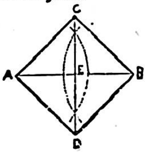


From B draw B D, perpendicular to A B and equal to it (P. 3.) From A and D as centres, *with* A B *as radius*, describe arcs cutting in C. Draw lines A C and D C, then

C A B D is the square required.

PROBLEM XXVI.

To construct a square, *having its diagonal* A B *given.*

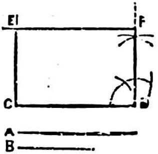


Bisect A B in E by the perpendicular C D. (P. 1 & 2.) From E cut off E C and E D, equal to E A and E B. Draw lines A C, A D, C B, and B D, then

A C B D is the square required.

PROBLEM XXVII.

To construct an oblong *or* rectangle, *having its adjacent sides equal to two given sides* A *and* B.



Draw a line C D equal to A. From D draw D F perpendicular to C D and equal to B. (P. 3.) From C as centre, *with* B *as radius*, and from F as centre, *with* A *as radius*, describe arcs cutting in E. Join C E and F E, then

E C D F is the oblong required.

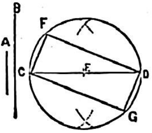
N.B.—*Be very careful to explain here the difference between an* OBLONG, RHOMBUS, RHOMBOID, and TRAPEZIUM.

PROBLEM XXVIII.

To construct an oblong, *having one side* A, *and its diagonal* B, *given.*

Draw a line C D equal to B. Bisect C D in E. (P. 1) From E as centre, *with* E C *as radius*, describe the circle C F D G. From C mark off on this circle C F equal to A, and from D, *on the other side of the diagonal*, cut off D G, also equal to A. Join C F, F D, D G, and C G, then

C F D G is the oblong required.

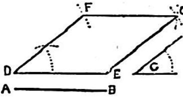


PROBLEM XXIX.

To construct a rhombus *having a base and angle equal to a given base* A B, *and angle* C.

Make a line D E equal to A B, and at D draw the line D F equal to D E, and making with it an angle equal to C. (P. 8.) From E and F as centres, *with* D E *as radius*, describe arcs cutting in G. Join F G and E G, then

F D E G is the rhombus required.

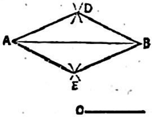


PROBLEM XXX.

To construct a rhombus, *having its diagonal* A B *and side* C *given.*

From A and B as centres, *with* C *as radius*, describe arcs cutting in D and E. Join A D, A E, B D, and B E, then

A D B E is the rhombus required.

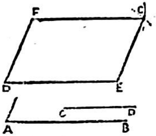


PROBLEM XXXI.

To **construct a rhomboid** *having its adjacent sides equal to two given lines* A B *and* C D, *and an angle equal to a given angle* A.

Draw a line D E equal to A B; from D draw D F equal to C D, and making with D E an angle equal to the given angle A. (P. 8.) From F, *with line* A B *as radius*, and from E *with line* C D *as radius*, describe arcs cutting each other in G. Join F G and E G, then

F D E G is the rhomboid required.

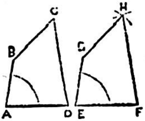


PROBLEM XXXII.

To **construct a trapezium** *equal to a given trapezium* A B C D.

Make E F equal to A D; from E draw E G equal to A B, and making with E F an angle equal to the angle B A D. (P. 8.) From G as centre, *with* B C *as radius*, and from F as centre, *with* C D *as radius*, describe arcs cutting in H. Join G H and F H, then

E G H F is the trapezium required.

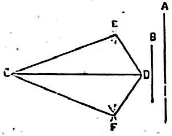


PROBLEM XXXIII.

To **construct a trapezium,** *having its adjacent pairs of sides equal respectively to two given lines* A *and* B, *and its diagonal equal to the given line* C D.

From centre C, *with the line* A *as radius*, and from D, *with* B *as radius*, describe arcs intersecting in E and F. Join C E, C F, D E and D F, then

C E D F is the trapezium required.



SECTION III.

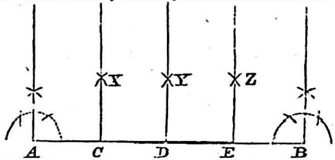
VARIOUS APPLICATIONS.

The foregoing problems include all the general principles that are required from First Grade pupils. The teacher, however, must not forget that they belong, as well, to the Second Grade course, and that the candidate will be required to apply them in solving various kindred problems, of which the following may be taken as fair illustrations.

PROBLEM XXXIV.

Make a line A B, 2 *inches long,* **divide it into four equal** *parts, and at each end and division* **erect a perpendicular** 1 *inch high.*

1. The pupil is familiar with all the lines of construction which here explain the process. The divisions may be made by the dividers or according to Problem 12.

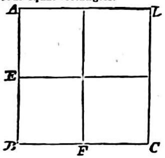


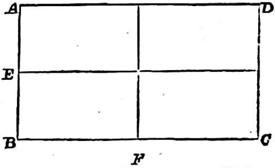
2.—The intersecting arcs at X are struck from A and D. The arcs at Y and Z are obtained in a similar manner. (See Prob. 2.)

PROBLEM XXXV.

Divide the given square A B C D *into* **four equal squares.**

Note.—The same methods may also be applied to the rectangle to divide it into four equal rectangles.



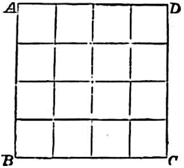


Bisect two adjacent sides A B and B C in E and F. Through E and F draw parallels to the other sides. Then the square is divided into four equal squares, and the rectangle into four equal rectangles.

N.B.—The parallels can be drawn either by set squares or after the method adopted in Problem 6.

PROBLEM XXXVI.

Divide the square A B C D *into* 16 (*or any other number*) equal squares.

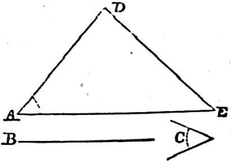


The method is apparent. Divide two adjacent sides into the number of equal parts, which are indicated by the square root of the number of equal squares required, which in this case is four.

Note.—The pupil should now be required to divide squares into 9, 25, 36, &c., equal squares.

PROBLEM XXXVII.

Construct a triangle, *two sides of which are equal to the two lines* A E *and* B *respectively, and one angle equal to* C.

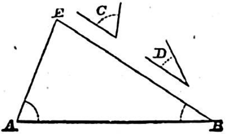


1. Make angle at A equal to the given angle C. (Prob. 8.)
2. Make the side A D equal to the given side B and join D E. Then

A D E is the triangle required.

PROBLEM XXXVIII.

Construct a triangle *having base* A B *and two angles, equal respectively to the two given angles* C *and* D.

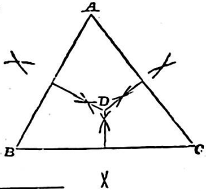


Make angles at A and B equal respectively to C and D. Produce the lines to meet in E. Then

A E B is the triangle required.

PROBLEM XXXIX.

Bisect each of the sides of the triangle A B C, *and produce the bisecting lines until they meet each other.*

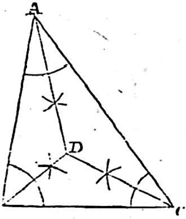


These sides are bisected as in Problem 1.

Note.—The point D, where the lines meet, is the centre of a circle, the circumference of which would pass through the points A, B, and C, and thus be described about the triangle. (See Prob. 64.)

PROBLEM XL.

Bisect each of the angles *of the triangle* **A B C,** *and produce the bisecting lines till they meet each other.*

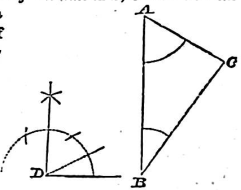


These angles are bisected as in Prob. 10. (Be careful to draw the arcs neatly.)

Note.—The point D where the bisecting line meets, is the centre of the triangle, from which a circle can be inscribed in the triangle. (See Prob. 63.)

PROBLEM XLI.

At the point **A**, *on the right side of the line* **A B, construct an angle of 60°,** *and at point* **B**, *on the same side,* **construct one of 30°.** *Produce the lines till they meet.*



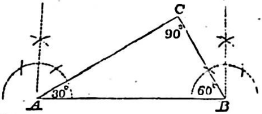
1st. Construct a right angle D and divide it into angles of 60° and 30°.

2nd. Apply these angles to A and B, as required, by means of Prob. 8.

PROBLEM XLII.

On the base A B **construct a triangle,** *the three angles of which are* 60°, 30°, *and* 90° *respectively.*

Note.—The three angles of every triangle are equal to two right angles, that is 180°.



Therefore, at A make an angle of 30°, and at B an angle of 60°, by lines A C and B C meeting in C. Then the remaining angle A C B will be 90°, and the three angles of the triangle will be respectively 60°, 30°, and 90°.

SCALE EXERCISES.

Note.—The teacher should frequently request his pupils to draw scales to represent various distances, such as the following:—

1.—Draw a line to represent $4\frac{1}{2}'$, on a scale of 1″ to the foot.

2.—Construct a scale to represent 25 miles, taking $\frac{1}{8}$th of an inch to the mile.

3.—Draw a line to represent 45 degrees, as marked on the side of a map, on a scale of 10 degrees to half an inch.

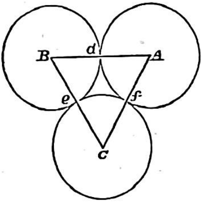
4.—Draw a line to represent 10′ on a scale of $\frac{1}{8}$th of an inch to an inch.

SECTION IV.

FIGURES TO BE COPIED.

This section chiefly tests the pupil's ability to measure, and to draw with neatness.

A.

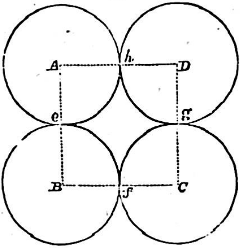


1st. Make the equilateral triangle in thick lines.

2nd. Mark off the points *d*, *e* and *f*, which will be the radii of the circles.

3rd. From centres A, B, C, with distances *d*, *e* and *f* respectively, describe the circles.

B.



1st. Make the square A B C D.

2nd. Mark off the points *e*, *f*, *g*, *h*, to give the radii of the circles.

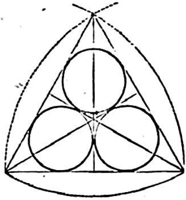
3rd. From centres A, B, C, D, with distances *e*, *f*, *g*, and *h* respectively, describe the circles, making the thick curves as shown.

C.

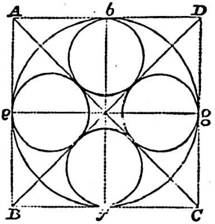
1st. Describe the triangle.

2nd. Draw the lines which bisect the angles meeting in the centre.

3rd. Mark off the centres and describe the circles, making the curves thick as required to show the outline of the *tre-foil*.



D.

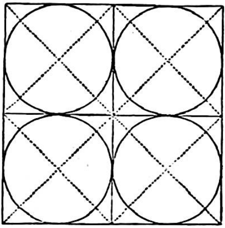


1st. Make the square A B C D, and draw the diagonals A C and B D.

2nd. Draw the lines $b\ f$ and $e\ g$, and mark off the centres of circles.

3rd. From each centre describe the circle, making the thick curves as shown.

E.



1st. Make the square.

2nd. Divide it into four equal squares.

3rd. Draw the dotted diagonals of the small squares.

4th. Describe the circles from the centres of each small square.

F.

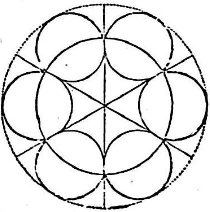
N.B.—The centre for the new figure would be given, if not, take any convenient point as centre.

1st. Describe the outside dotted circle.

2nd. Describe the inner circle.

3rd. Draw the three diameters.

4th. Mark off the centres of the small circles and describe them.



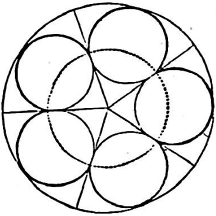
G.

N.B.—The centre for the new figure would be given, if not, take any convenient point as centre.

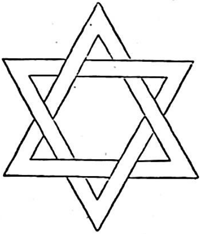
1st. Describe the outside circle and then the inside one.

2nd. Draw the radii and mark off the five centres.

3rd. From each centre describe the circles, making the thick curves as shown to mark the outline of the *cinque-foil.*



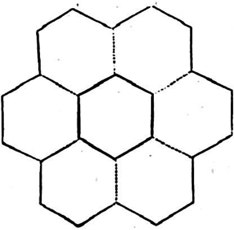
H.



Note.—This figure consists of two equilateral triangles.

The construction merely requires exact measurement and careful ruling.

I.



1st. Make the inside hexagon with thick lines as shown.

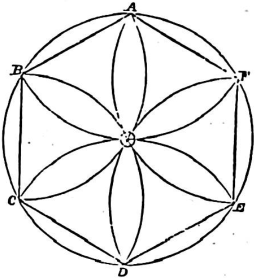
2nd. On each side describe a hexagon and the figure will be complete.

J.

N.B.—The centre for the new figure would be given, if not take any convenient point as centre.

1st. Describe the circle and inscribe the lines of the hexagon.

2nd. From centre A, with distance A O, describe the arc B O F. From each of the other angular points of the hexagon, with the radius of the circle as distance, describe similar arcs.



K.

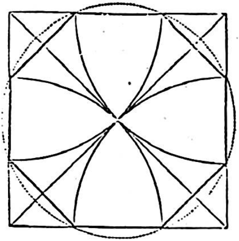
1st. Draw the square and its diagonals.

2nd. Mark off the points which trisect the sides.

3rd. From each angle of the square as centre describe the arcs.

4th. Describe the circle.

5th. Mark the thick lines to construct the octagon.



SECTION V.

The Problems contained in the following Sections belong exclusively to the Second Grade course; nevertheless, the foregoing Problems have to be known by the candidates.

OFFICIAL INSTRUCTIONS.

The instructions issued by the Science and Art Department say :—

"Problems will be set in the elementary constructions necessary "to geometrical *pattern-drawing,* and simple *geometrical tracery* "—the constructions for a *circle passing through three points* or "*touching three lines;* the construction of *tangents* to two *circles;* "the simple cases of *inscription* and *circumscription;* the *reduction* "and *enlargement of figures;* the construction of *irregular polygons* "when the angles and sides are given ; the use of *plane scales;* and "of the *scale of chords;* the construction of *regular polygons.* (In "this stage a general method for the construction of polygons will "be considered sufficient.) In Solid Geometry, simple Problems "on the *plan, elevation* and *section* of the *common solids,* and the "projection of *plane figures,*" will be given.

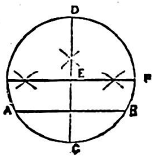
CIRCLES, AND THEIR TANGENTS.

PROBLEM XLIII.

To find the centre *of a circle.*

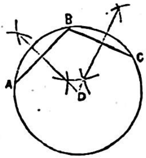
Draw any chord A B and bisect it by a perpendicular C D which will be a diameter of the circle. Bisect C D in E; then

E is the centre of the circle.



PROBLEM XLIV.

To draw a circle *through three given points,* A, B, C.



Join A B and B C, and bisect the lines by perpendiculars, cutting in D. From D as centre, *with* D A *as radius,* describe a circle. It will pass through the three given points A, B, C; then

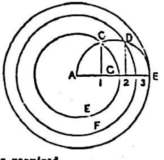
A B C is the circle required.

Note.—An arc of a circle can be drawn through any three points, not in the same straight line, as well as a whole circle. The centre of a circle, of which any arc is a part can also be found by bisecting any two adjacent chords, the meeting point of the lines of bisection being the centre.

PROBLEM XLV.

To divide *the area of a circle into any number of equal or proportional areas by concentric circles.*

Let it be required to divide the area of the circle into three equal parts.



Let A be the centre of the circle. Draw any radius A B, and divide it into as many equal parts as the circle is to have divisions (in this case 3.) Upon A B describe a semicircle, and erect perpendiculars, from 1 and 2 cutting it in C and D. From A as centre, *with* A C *and* A D *as radii,* describe the circles C E and D F; then the area of the given circle is divided into three equal divisions as required.

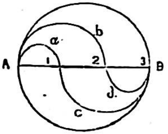
N.B.—If it is required to divide the area of the circle into *proportional areas* by concentric circles, the radius A B must be divided into the *proportional parts* required instead of equal parts.

PROBLEM XLVI.

To divide *the area of a circle into any number of equal parts which shall also have equal perimeters.*

Let it be required to divide the area of the circle into three equal parts.

Draw any diameter A B, and divide it into as many equal parts as the figure is to have divisions (in this case 3.) Upon A 1 and A 2, describe the semicircles *a* and *b*, and upon B 1 and B 2, describe the contiguous semicircles *c* and *d*; then the circle is divided into three equal parts by the double semicircles *a c* and *b d*.



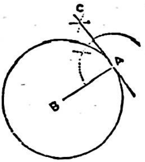
TANGENTS.

PROBLEM XLVII.

To draw a tangent *to a given circle at a given point of contact* A

Find the centre B. Join B A. From A, draw A C perpendicular to A B, and produce it; then

A C is the tangent required.

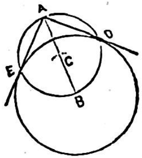


PROBLEM XLVIII.

To draw a tangent *to a given circle from a given point* A *outside it.*

Find the centre B Join A B, and bisect it in C. From C as centre, *with* C A *as radius*, describe a circle, cutting the circumference in D and E. Join A D and A E, and produce the lines; then

both the lines A D and A E are tangents to the circle.

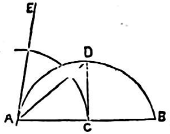


PROBLEM XLIX.

To draw a **tangent** *to the arc of a circle at a given point* **A**, *without using the centre.*

Draw the Chord A B and bisect it in C. From C erect perpendicular C D, and join A D. Make the angle D A E equal D A C; then

E A produced is the tangent required.



QUESTIONS AND EXERCISES.

1.—Find the centre of any given circle.

2.—Through any given point C within a circle, whose radius is 1½", draw the longest possible chord.

3.—Construct the plan of a circular race course, 1¼ mile in diameter, so that three gates A, B, and C shall fall within the path. (*Scale 1" to a mile.*)

4.—Construct the circle of which any arc is a part.

5.—Divide the area of a given circle into eight equal sectors, by lines drawn from the centre.

6.—Construct a plan to show how any given circular estate could be divided equally between four farmers A, B, C, and D, so that A's estate is entirely enclosed by B's; B's estate by C's; and C's estate by D's.

7.—Construct a plan to show how any circular flower-bed could be divided into four equal parts, so that the boundaries of each part are equal.

8.—From a given point C, one inch outside the circumference of a given circle whose diameter is two inches, draw a tangent to the circle.

9.—Divide a circle into three proportional areas by means of concentric circles, so that the area of the outside circle is three times that of the inside one, and the middle area twice that of the inside one.

10.—Construct a circle touching another circle in A, the circumference also passing through two given points B and C, either without or within the circle.

11.—Draw a tangent touching an arc in any given point C, without using the centre.

SECTION VI.

POLYGONS.

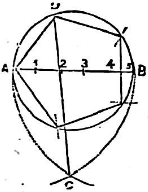
"A general method for the construction of Polygons will be considered sufficient."—SEE INSTRUCTIONS.

PROBLEM L.

To inscribe any regular Polygon *in a given circle.*

GENERAL METHOD.

For example, let the required Polygon be a Pentagon.



Draw a diameter A B and divide it into as many equal parts as the figure is to have sides in this case 5. From A and B as centres, *with* A B *as radius,* describe arcs cutting in C. From C draw the line C D, passing through the second* division from A, cutting the circumference in D. Join A D, which will be one of the sides, or nearly so, of the Pentagon. Set off the distances A D around the circle to get the points for the angles of the Polygon. Join these points, then

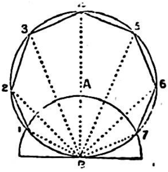
the figure inscribed in the circle is the Polygon required. (In this case a Pentagon.)

PROBLEM LI.

To inscribe any regular Polygon *in a given circle.*

SECOND METHOD.—(EXTRA.)

For example, let the required Polygon be an Octagon.



Draw any radius A B. At B draw a tangent to the circle (P. V.) From B with any radius describe a semicircle, and divide it into as many equal parts as the Polygon is to have sides, (in this case 8.) Draw radiating lines from B through each of these divisions, cutting the given circle in 1, 2, 3, 4, 5, 6, 7. Join these points; then

the required Polygon (an Octagon) is inscribed.

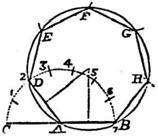
* Always draw the line as C D through the second division, whatever number of sides the Polygon is to have.

PROBLEM LII.

To describe any regular Polygon *on a given line* A B.

GENERAL METHOD.

For example, let the required Polygon be a Heptagon.



Produce A B either way to C, and from A as centre, with A B as radius, describe a semicircle, cutting the produced line in C. Divide this semicircle into as many equal parts as the figure is to have sides (in this case 7.) Join A and D, the second* point from C, which will be another side of the heptagon. Through the points B, A, D, describe a circle (P. II.) Set off the distance A D around the circumference, and join the parts thus marked, viz., D E, E F, F G. G H, and H B: then

A D E F G H B is the Polygon required. (In this case a Heptagon.)

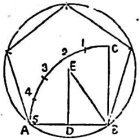
The pupil may omit the second and special methods for constructing Polygons and pass on to Problem LXI.

PROBLEM LIII.

To describe any regular Polygon *on a given line* A B.

SECOND METHOD.—(EXTRA.)

For example, let the required Polygon be a Pentagon.

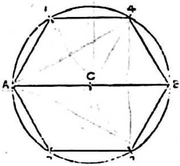


At B erect a perpendicular B C equal to A B and describe the quadrant A C. Divide the arc A C into as many equal parts as the required polygon is to have sides, (in this case 5.) Draw a line from B to the 2nd division. Bisect A B in D, and from D erect a perpendicular to cut B 2 in E. From centre E with distance E B describe a circle, it will contain the Polygon. Mark off A B around it, then the required Pentagon is described on A B.

* To get the second side always draw the line as A D to the second division, whatever number of sides the Polygon is to have.

PROBLEM LIV.—(EXTRA.)

To inscribe a **hexagon** *in a given circle.*



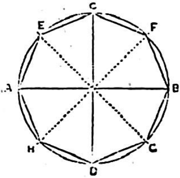
Draw any diameter A B. From A and B as centres, with the radius of the circle as distance, cut off the points 1 2, 3 and 4. Draw lines A 1, A 2, B 3, B 4, 1 4, and 2 3; then

the inscribed figure is the Hexagon required.

Note.—The radius of a circle is a sixth part of the circumference (or nearly so.)

PROBLEM LV.—(EXTRA.)

To inscribe an **octagon** *in a given circle.*

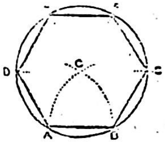


Draw any two diameters A B and C D, at right angles to each other; bisect each of the four arcs by the diameters E G and F H. Join the points; then the figure inscribed in the circle is the required Octagon.

N.B.—The special methods for inscribing other Polygons in a circle are more difficult than either of the general methods (Problems L. and LI.), therefore it is quite unnecessary to give them here.

PROBLEM LVI.—(EXTRA.)

To describe a **hexagon** *on a given line* **A B.**



From A and B as centres, *with* A B *as radius* describe arcs cutting in C. From C as centre; *with the same radius*, describe a circle, it will contain the required hexagon. Cut off around the circumference, with A B as radius, the points D, E, F, and G; they will be the corners of the hexagon. Join these points; then

the figure A D E F G B is the hexagon required.

Note.—The radius of a circle is the length of the side of the Hexagon inscribed in it.

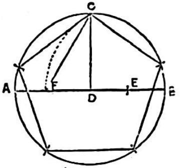
PROBLEM LVII.

To inscribe a Pentagon *in a given circle.*

SPECIAL METHODS.

Draw any diameter A B and a radius D C perpendicular to it. Bisect B D in E. From E, with E C as radius, describe arc C F. Join F C, it will be equal to the side of the pentagon. From C mark off the five distances equal to F C, and join the points; then

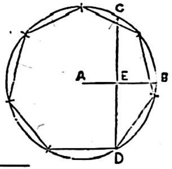
the inscribed figure is the Pentagon required.



PROBLEM LVIII.

To inscribe a Heptagon *in a given circe.*

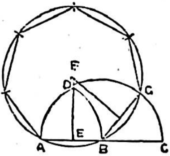
Draw any radius A B, bisect it by the perpendicular C D in E, then E C will be equal to the side of the heptagon. Inscribe it.



PROBLEM LIX.

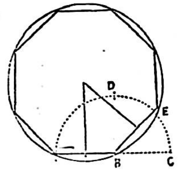
To describe a Heptagon *on a given line* A B.

From B as centre, with radius A B, describe a semicircle cutting A B produced in C. From A, with same radius, cut this semicircle in D. Bisect A B in E, and join D E. From C, with D E as radius, cut the semicircle in G. Join B G, it is another side of the Heptagon. Find the centre of the circle that contains it, and complete the Hepagon.



PROBLEM LX.

To describe an Octagon *on a given line* A B.



From B, with radius A B. describe a semicircle cutting A B produced in C. Bisect the semicircle in D, and bisect the arc C D in E. Draw line B E, it is another side of the octagon. Find the centre of the circle that contains it and complete the octagon.

N.B.—The special methods for describing any of the other polygons on a given base are more difficult than either of the general methods, therefore it is quite unnecessary to give them here.

TO CIRCUMSCRIBE POLYGONS.

GENERAL REMARKS.

First.—The centre of any regular polygon is the centre of the circle which circumscribes it.

Second.—The angles at the centre of any regular polygon are equal to each other, and they are together equal to four right angles.

Therefore any regular polygon can be inscribed in a circle by using the protractor only. (*See Questions* 5, 7, 9, *and* 10 *at the end of this chapter.*)

Third.—If tangents be drawn to the angular points of any polygon inscribed in a circle, a polygon will be described about the circle having the same number of sides. (*See Prob. LXI.*)

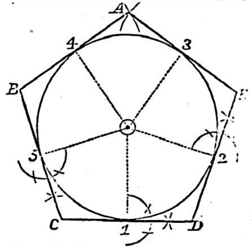
Fourth.—If tangents be drawn to a circle parallel to the sides of any inscribed polygon, a polygon will be described about the circle having the same number of sides.

Therefore any polygon can be easily described about a circle.

Second Grade Pupils are required to describe Polygons about circles as well as to inscribe them in circles.

PROBLEM LXI.—GENERAL METHOD.

To describe any regular polygon. *about a given circle. Let the required Polygon be a Pentagon.*



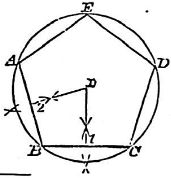
The pupil must divide the circumference of the circle into as many equal parts as the Polygon is to have sides in this case 5, and at these point draw tangents. The Circle may be divided as in Problem XL., or it may be done more neatly by constructing five equal angles at the centre. Now, since the angles at the centre of every Polygon are equal to four right angles (360°) therefore the angles at the centre of a Pentagon are each 72°. Find the centre O and draw any radius. At O in O 1 make an angle of 72° by Protractor and in the same manner 4 other equal angles in consecutive order by the radii, O 2, O 3, O 4 and O 5. Draw tangents at these points to intersect each other in A B C D E. Then

A B C D E is the Pentagon required.

Second Grade Pupils are also required to complete Polygons when two or three sides are given.

PROBLEM LXII.—(GENERAL METHOD.)

Complete the Polygon *of which A B and B C are the adjacent sides.*



The Pupil will notice that this is only a part of Problem VIII. Since every regular Polygon can be inscribed in a circle, we have simply to find the centre of the circle and draw it through the three points A B and C. Bisect A B and B C in 2 and 1. At 2 and 1 erect perpendiculars meeting in X, which is the centre of the circle. Describe the circle and complete the Polygon, which is a Pentagon.

TO CONSTRUCT EQUIVALENT IRREGULAR POLYGONS.

Second grade pupils are also required to draw irregular figures similar and equal to others.

The process is simple. Start with any one side and make a side equal to it, construct an angle at one end equal to the angle at the corresponding end, and make the

adjacent side equal to the corresponding side, and so on, make an equal angle and equal side, corresponding with the given figure, until the irregular figure is complete.

Note.—It will be well for the pupil to draw various irregular rectilineal figures at random, and construct equivalent ones.

QUESTIONS AND EXERCISES.

1.—In a given circle whose diameter is 1¼ inch, inscribe a regular pentagon in two different ways.

2.—Show the position of a wheat sheaf situated exactly in the middle of a corn field, which is bounded by seven equal hedges.

3.—Draw a circle which shall pass through three consecutive angles of any regular nonagon.

4.—Divide a regular heptagonal flower bed into seven equal divisions, so that each division shall have one side of the heptagon.

5.—Prove by illustrations that the angles made by straight lines drawn from the centre of any polygon to the angular points, are together equal to four right angles.

6.—How can a polygon of any number of sides, be inscribed in a circle by means of the protractor only?

7.—Construct a regular polygon whose side A B is the chord of an arc of 45°.

8.—What is the number of degrees in each of the angles at the centre of a duodecagon?

9.—Inscribe in any given circle, that polygon whose angles at the centre are each 60°.

10.—Inscribe in any given circle, that irregular hexagon whose angles at the centre are respectively 120°, 90°, 60°, 45°, 30, and 15°.

11.—Construct a regular polygon on any given line A B, having the distance from either of its extremities to the centre equal to the side A B.

12.—About any given circle, describe a regular pentagon, whose sides are parallel to an inscribed pentagon.

13.—About a given circle, describe a regular heptagon, whose sides are drawn through the angular points of an inscribed heptagon.

14.—Draw as many illustrations as you can, showing how triangles, quadrilaterals, circles and polygons, are familiarly applied in giving beauty to objects of *design, ornament, architecture, &c.*

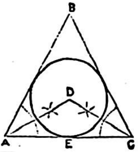
SECTION VII.

INSCRIBED & DESCRIBED FIGURES.

PROBLEM LXIII.

To inscribe a circle *in a given triangle* A B C.

Bisect the two angles B A C and A C B by lines meeting in D. From D draw D E perpendicular to A C. From centre D, with radius D E, inscribe the required circle.

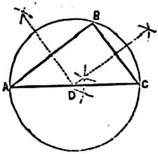


PROBLEM LXIV.

To describe a circle *about a given triangle* A B C.

Bisect two of its sides, A B and B C, by lines cutting in D. From D as centre, *with* D B *as radius*, describe a circle; then .

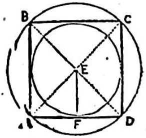
A B C is the required circle described about the given triangle.



PROBLEM LXV.

To inscribe or describe a circle *in or about a given square* A B C D.

Draw the diagonals A C and B D, intersecting in E. From E draw E F perpendicular to A D. From E as centre, with E F as radius, draw a circle which will be *inscribed* in the square; also from E as centre, *with* E C *as radius*, draw a circle which will be *described* about the square.

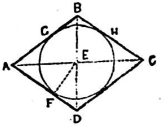


PROBLEM LXVI.

To inscribe a circle *in a given rhombus* A B C D.

Draw the diagonals A C and B D, intersecting in E. From E draw E F perpendicular to A D from centre E, with *radius* E F, describe the circle F G H; then

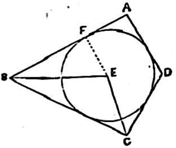
F G H is the required circle inscribed in the given rhombus.



PROBLEM LXVII.

To inscribe a circle *in a* trapezium A B C D, *which has its adjacent pairs of sides equal.*

Bisect any two of its adjacent angles, as A B C and B C D, by lines meeting in E. From E draw E F perpendicular to one of the sides. Then the point E will be the centre, and E F the radius of the required circle. Describe it.

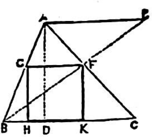


PROBLEM LXVIII.

To inscribe a square *in a given triangle* A B C.

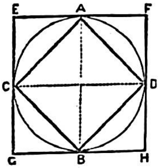
From A draw A D perpendicular to B C and A E perpendicular to A D and equal to it. Draw the line E B, cutting A C in F. Through F, draw F G parallel to B C, and through F and G draw the lines F K and G H parallel to A D; then

F G H K is the square required.



PROBLEMS LXIX. AND LXX.

To inscribe or describe a square *in or about a given circle.*



Prob. LXIX.—*To inscribe a square in a circle.*
Draw any two diameters, A B and C D, at right angles to each other (Prob. XLIII.) Join the ends of these diameters; then

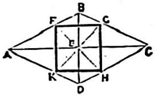
A C B D is the square required, inscribed in the circle.

Prob. LXX.—*To describe a square about a circle.*
At the points A, B, C, and D, draw tangents, meeting in E, G, H, and F, (P. XLVII.); then

E G H F is the square required, described about the circle.

PROBLEM LXXI.

To inscribe a square *in a given rhombus* A B C D.

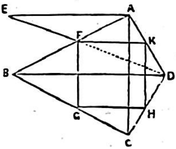


Draw the diagonals A C and B D, intersecting in E. Bisect the angles A E D and A E B by lines produced each way, to cut the sides of the rhombus in K, F, G, and H. Join F K, K H, H G, and G F; then

F K H G is the required square, inscribed in the given rhombus.

PROBLEM LXXII.

To inscribe a square *in a given trapezium* A B C D *which has its adjacent pairs of sides equal.*



Draw the diagonals A C and B D. From A draw A E perpendicular to A C, and equal to it. Draw line E D cutting A B in F. From F draw F G parallel to A C; and through F and G draw F K and G H parallel to B D. Join K H; then

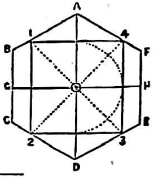
F G H K is the required square, inscribed in the given trapezium.

PROBLEM LXXIII.

To inscribe a square *in a given hexagon* A B C D E F.

Draw the diagonal A D; bisect it in O, and draw the diameter G H perpendicular to it. Bisect the adjacent angles G O A, and A O H, by lines produced to cut the sides of the hexagon in 1, 2, 3, 4. Join these points; then

1 2 3 4 is the required square.

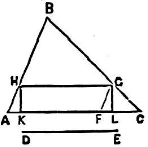


PROBLEM LXXIV.

To inscribe an oblong *in a given triangle* A B C, *having a side equal to a given line* D E.

On A C set off A F equal to D E. Through F draw F G parallel to A B, and through G draw G H parallel to A C. From G and H, draw G L and H K, perpendicular to the base; then

H K L G is the required oblong, inscribed in the given triangle, A B C.

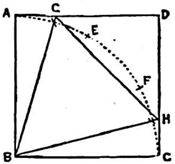


PROBLEM LXXV.

To inscribe an equilateral triangle *in a given square* A B C D.

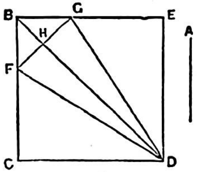
From B, with A B as radius, describe the quadrant A C. From A and C, with same radius, cut off A F and C E. Bisect A E and C F, and through the points of bisection draw lines B G and B H, cutting the sides of the square in G and H. Draw line G H; then

G B H is the equilateral triangle required, inscribed in the given square.



PROBLEM LXXVI.

To inscribe an isosceles triangle *in a square* B C D E, *having a base equal to a given line* A.



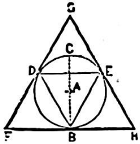
Draw the diagonal B D. From B cut off B H equal to half the line A. Through H draw line F G perpendicular to B D cutting the sides of the square in F and G. Join D F and D G; then

D G F is the isosceles triangle required.

PROBLEMS LXXVII AND LXXVIII.

To inscribe or describe an equilateral triangle, *in or about a given circle* B D E.

Prob. LXXVII.—*To inscribe an equilateral angle* in *the circle* B D E.



Find the centre A, and draw any diameter B C. From C as centre, *with* C A *as radius*, cut the circle in D and E. Join E B, B D, and D E; then

B D E is the required equilateral triangle, inscribed in the circle B D E.

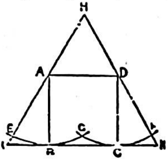
Prob. LXXVIII.—*To describe an equilateral triangle* about *the circle* B D E.

At B, D, and E draw tangents (P. XLVII.) meeting in F, G, and H; then

F G H is the required equilateral triangle, described about the circle B D E.

PROBLEM LXXIX.

To describe an equilateral triangle *about a given square* A B C D.

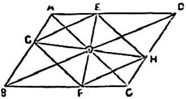


From points A and D, *with* A B *as radius*, describe arcs cutting in G. From G as centre, with same radius cut these arcs in E and F. Join A E and D F, and produce them to meet in H. Produce B C until it cuts the lines H E and H F produced in I and K; then

H I K is the required equilateral triangle described about the square A B C D.

PROBLEM LXXX.

To inscribe a four-sided equilateral figure *in any parallelogram* A B C D.



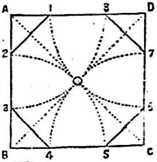
Draw the diagonals A C, B D, and bisect any two of the adjacent angles by lines cutting the sides of the parallelogram in E, G, F, H. Join these points; then

E G F H is the quadrilateral figure required, inscribed in the given parallelogram.

N.B.—If it is required that the inscribed figure should touch the parallelogram at any given point as at G, draw a line from G through the centre, and another diameter at right angles to it. The points where these lines touch the parallelogram will be the angular points for the inscribed figure required.

PROBLEM LXXXI.

To inscribe an octagon *in a given square* A B C D.

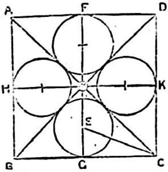


Draw the diagonals A C and B D, intersecting in O, and from each of the angles of the square, with A O as radius, cut off the 8 points as marked. Join these points; then

1 2 3 4 5 6 7 8 is the required octagon inscribed in the given square A B C D.

PROBLEM LXXXII.

To inscribe four equal circles *in a given square* A B C D *touching each other and one side only of the square.*

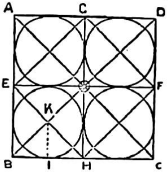


Draw diagonals A C and B D which will divide the square into the four equal triangles A O D, A O B, B O C, and C O D. In each triangle inscribe a circle by Problem LXIII, touching the sides of the square in H, G, K, and F; then

the four circles required are inscribed in the square A B C D.

PROBLEM LXXXIII.

To inscribe four equal circles *in a given square* A B C D, *each circle touching two of its sides.*

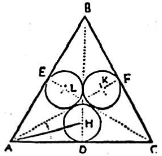


Bisect any two sides as A B and A D in E and G, and draw the diameters E F and G H. The given square is now divided into four equal squares. In each of these squares inscribe a circle by Problem LXV., and the

four circles required will be inscribed in the given square.

PROBLEM LXXXIV.

In a given equilateral triangle A B C, *to inscribe* three equal circles *touching each other and one side of the triangle.*



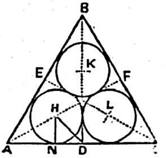
Bisect each side of the triangle and draw the lines D B, E C, and F A. Bisect the angle F A C by a line cutting B D in H. From E and F cut off E L and F K equal to D H; then

H, K, and L are the centres of the three circles required, of which H D is the radius.

N.B.—*Six equal circles* can be inscribed in this triangle if we draw lines parallel to the sides of the triangle through H, L, and K. The three angular points of the equilateral triangle thus made, will be the centres for the other three circles.

PROBLEM LXXXV.

In a given equilateral triangle A B C, *to inscribe* three equal circles *touching each other and two sides of the triangle.*



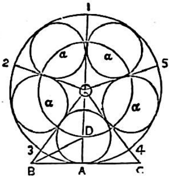
As in Problem LXXXIV., bisect the sides of the triangle and draw the lines D B, E C, and F A; but bisect the angle A D B (instead of F A C) by a line cutting A F in H. From H draw H N perpendicular to A C, and from B and C, cut off B K and C L equal to A H; then

H, L, and K are the centres of the three circles required, of which H N is the radius.

PROBLEM LXXXVI.

To inscribe any number of equal circles *in a given circle. Take for example,* 5.

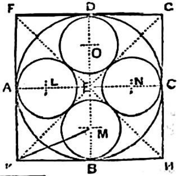
GENERAL METHOD.



Divide the circumference of the circle into as many equal divisions as the number of circles required, (in this case 5,) as 1, 2, 3, 4, 5. Find the centre O, and draw the radii O 1, O 2, O 3, O 4, and O 5. In each of these 5 sectors, a circle is to be inscribed. Bisect either sector as 3 O 4, by radius O A, and draw a Tangent at A (P. V.) cutting O 3 and O 4 produced in B and C. Find D the centre of this triangle, (P. LXIII) and inscribe a circle in it. From O as centre, with O D as radius, describe a circle. Mark off on this circle from D the centres for the four other circles as *a*, *a*, *a*, *a*, and inscribe them.

PROBLEM LXXXVII.

In a given circle A B C D to inscribe four equal circles.

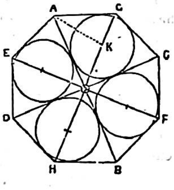


Draw the diameters A C and B D cutting each other at right angles (Problem I.) Draw tangents at A, B, C, and D, meeting in F, G, H, and K (P. LXVII.) Join F H and G K. Bisect the angle G K H by line cutting B D in M. From C, D, and A cut off C N, D O, and A L, equal to B M; then

M, N, O. and L are the centres of the four circles required, of which B M is the radius.

PROBLEM LXXXVIII.

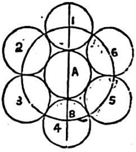
To inscribe four equal circles *in a given octagon.*



Draw any two diagonals at right angles to each other as A B and C D intersecting in the centre O. The octagon is now divided into four equal trapezii; find the centre of each trapezium and inscribe a circle (P. LXVII.)

PROBLEM LXXXIX.

To describe six equal circles *about, and equal to a given circle, touching each other and the given circle.*



From the centre A of the given circle, and with its diameter as radius, describe the circle 1, 2, 3, 4, 5, 6. Draw the diameter 1 A 4, and from 1 with the radius of the given circle describe a circle touching it. Mark off the other centres 2, 3, 4, 5, 6. From these points, with the radius of the given circle, describe the remaining circles, which will touch each other and the given circle.

Note.—To inscribe these circles in another circle is a simple deduction from this.

QUESTIONS AND EXERCISES.

1.—In and about any given triangle inscribe and describe a circle.

2.—In any given circle inscribe a square.

3.—Inscribe in any given circle a triangle that shall cut off equal segments.

4.—In a given circle, 2″ in diameter, inscribe seven equal circles, six of which shall touch the given circle and a central one. (See Problem LXXXIX.)

5.—In any scalene triangle whose base is 2 inches inscribe an oblong whose base is 1½ inch.

6.—About any given square describe a triangle whose sides are equal.

7.—In any given square sufficiently large inscribe a triangle having its two sides equal and its base 1 inch long.

8.—In any hexagon inscribe a square.

9.—In any octagon inscribe four equal circles.

10.—In any given circle inscribe 3, 5, and 7 equal circles.

11.—In any equilateral triangle inscribe 6 equal circles. (Refer to Problem LXXXIV., in which the method is described.)

12.—In any given square inscribe a regular polygon that shall cut off four equal corners of the square.

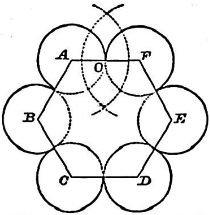
SECTION VIII.

FOILED FIGURES.

PROBLEM XC.

To construct a foiled figure *about any regular polygon having tangential arcs. Say a hexagon* A B C D E F.

THE HEXA-FOIL.



Bisect one side A F in O. From each of the angular points, A, B, C, D, E, F, with radius A O, describe the arcs as illustrated in drawing.

N.B.—These arcs are tangential because they do not cut each other, and thus they differ from the arcs in the foiled figures 91, 92, and 93, which are not tangential, but have adjacent diameters.

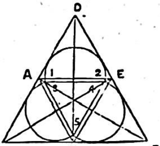
If the foiled figure is to have a given radius, construct the polygon having sides equal to twice that radius.

N.B.—The Gothic *tre-foil, quatre-foil,* and *cinque-foil* are constructed on this method

PROBLEM XCI.

THE TRE-FOIL.

To inscribe, within a given equilateral triangle, three equal semicircles, *having their adjacent diameters equal.*

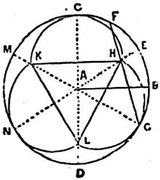


Bisect the angles of the triangle by lines D C, F E, and B A, and inscribe the triangle A E C. On A E describe the semicircle 1, 2, touching the sides of the triangle. Through 1 and 2 draw parallels to A C and E C meeting C D in 5, and cutting A B and E F in 3 and 4. Join 3 and 4; then the sides 3 4, 3 5, and 4 5 are the adjacent diameters of the semicircles—complete them

PROBLEM XCII.

To inscribe, within a given circle, three equal semicircles *having adjacent diameters.*

Find centre A (P. 43.) Draw any diameter C D, and the radius A B perpendicular to it. Trisect the arc B C in E and F.
On the other side of B cut off B G equal to B E. Draw the diameters G M and E N. Join F G, cutting the diameter E N in H. From centre A, with distance A H, cut off K and L on the diameters M G and D C. Join H, K, and L; then



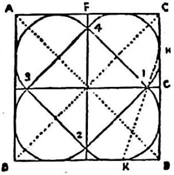
H K, K L, and L H are the adjacent diameters of the three semicircles —describe them.

PROBLEM XCIII.

THE QUATRE-FOIL.

To inscribe four equal semicircles *in a given square having adjacent diameters.*

Draw the diagonals A B and C D, and the central diameters F and G. Bisect G C in H, and from B cut off B K equal to G H. Join K H, by a line cutting the diameter G in 1. Set off distance G 1 from each extremity of the diameters as 2, 3, 4. Join the points 1, 2, 3, 4; then

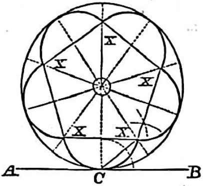


the lines thus drawn are the adjacent diameters of the four semicircles— describe them.

N.B.—*Foiled figures* of different kinds are commonly used in window tracery of Gothic Architecture. The *tre-foil* has three lobes like the clover leaf, the *quatre-foil* four lobes, the *cinque-foil* five lobes, the *hexa-foil* six lobes, &c.

PROBLEM XCIV.—GENERAL METHOD.
THE CINQUE-FOIL.

To inscribe within a given circle *any number of equal semicircles having adjacent diameters, say* 5.



Divide the circumference of the circle into as many equal parts as the number of semicircles required, in this case five, and from each of these points draw diameters. At any point, as C, draw a tangent A C B and bisect the angle O C B by line C X, which cuts the diameter in X. On each alternate semi-diameter set off the distance O X from O. Join the points marked X and the lines obtained will be the adjacent diameters of the required semicircles. The points of intersection of these diameters, with the diameters of the circle, will give the centres of the semicircles required which can now be described

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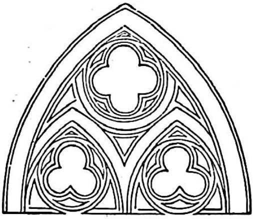
The following figure illustrates a common application of the Tre-foil and Quatre-foil in ornamenting Gothic windows.



QUESTIONS AND EXERCISES.

1. Construct a *tre-foil*, *quatre-foil*, and *cinque-foil*, having tangential arcs, the radius of which is $\frac{1}{2}$ inch.

2. Describe a *tre-foil*, *quatre-foil*, and *cinque-foil*, having adjacent diameters of $\frac{3}{4}$ inch.

3. Describe a circle having a radius of $1\frac{1}{2}$ inch, and in it inscribe a trefoil having adjacent diameters.

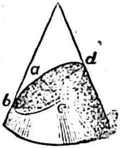
SECTION IX.

THE ELLIPSE.

The Ellipse is one of the three conic sections, by which we mean the sections obtained by cutting a cone.

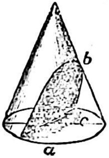
THE ELLIPSE.

Fig. 1.



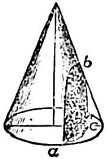
THE PARABOLA.

Fig. 2.



THE HYPERBOLA.

Fig. 3.



The base of a cone is a circle, and if a slice be cut from a cone parallel to the base, the base of the portion cut off would also be a circle. The portion remaining is called a *Truncated Cone*, or *Frustrum.*

The Ellipse.—If a cone be cut off by an oblique plane the shape of the section is an Ellipse, as *b c d*, Fig. 1. An Ellipse has two unequal diameters or axes which bisect each other at right angles. The long diameter, as *b a*, is called the *Transverse diameter*, and the short diameter, the *Conjugate diameter.*

Parabola.—If a cone be cut off by a plane which is parallel to the sloping edge of the cone, the shape of the section is a *Parabola*, as *a b c*, Fig. 2.

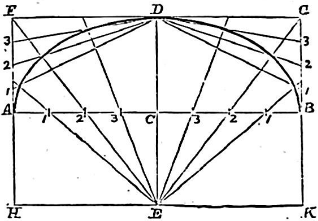
Hyperbola.—If a cone be cut off by a plane which is perpendicular to the base, that is, parallel to its axis, the shape of the section so cut is an *Hyperbola*, as *a b c*, Fig. 3.

The Oval.—An oval differs from an ellipse by being *egg-shaped;* that is, one of the ends is larger than the other.

PROBLEM XCV.

To describe an ellipse *by means of intersecting lines, the transverse and conjugate diameters,* A B *and* D E *being given.*

FIRST METHOD.

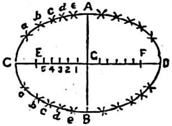


Let A B and D E be the tranverse and conjugate diameters. Through A and E draw lines parallel to D E, and through D and E draw lines parallel to A B, all meeting in F, H, G, K. Divide A F and G B into any number of equal parts, say 4, and divide A C and C B into the same number of equal parts. Join D and the points 1, 2, 3, in A F and B G; and join E and the points 1, 2, 3, in A C and B C, and produce the lines to cut those drawn from D. The points of intersection, as shown in the figure, will be the true points for the curve of one-half of the ellipse. In a similar manner find points for the other half.

PROBLEM XCVI.

To describe an ellipse *by means of intersecting arcs, the transverse and conjugate diameters,* C D *and* A B *being given.*

SECOND METHOD.—(THE FOCI.)



Let the diameters bisect each other in G, at right angles. From A or B, with C G as radius, cut C D in E and F.

These points are the Foci of the Ellipse.

Divide G E and G F, the distances between the Foci and the centre, into any number of parts as 1, 2, 3, 4, 5; they may be equal.

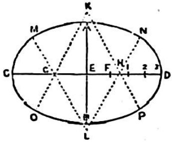
but it will be better if the distances decrease as they approach the Foci. From E and F with the radius D 5 and C 5, D 4 and C 4, D 3 and C 3, D 2 and C 2, D 1 and C 1, describe arcs intersecting on each side of the diameter A B in *a*, *b*, *c*, *d*, and *e*. Through the points of intersection draw the curve of the ellipse by hand.

Note.—This method is a better one than the first method, Prob. XCV.

PROBLEM XCVII.

To construct an ellipse *with arcs of circles, the diameters* A B *and* C D *being given.*

THIRD METHOD.

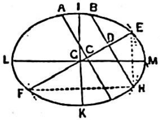


Let the diameters A B and C D bisect each other in E at right angles. From C, with A B as radius, mark the point F. Divide the distance F D into three equal parts. From E, with two of these parts as radius, cut line C D in H and G. From H and G, with H G as radius, describe arcs intersecting in K and L. From K and L, with radius K B, describe arcs O P and M N, and from H and G, with H D as radius, describe arcs N P and M O to complete the ellipse. Lines drawn from K and L through G and H will show where the four arcs unite.

Note.—If required *to inscribe an ellipse in a given rectangle*, bisect two of the adjacent sides and draw parallels for the two diameters of the ellipse; then proceed as in Prob. XCV.

PROBLEM XCVIII.

To find the centre *and* axes *of an ellipse.*



Draw any two parallel chords A and B and bisect them in C and D. Draw a diameter E F through D and C and bisect it in G, then

G is the centre of the ellipse.

From G, *with* G E *as radius*, mark the point H. Join E H and H F; through G draw I K and L M parallel to E H and F H; then

L M and I K are the axes required, or the Transverse and Conjugate diameters.

PROBLEM XCIX.

To complete an ellipse *from a small arc.*

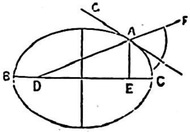
(A deduction from the last Problem.)

Sometimes a small portion of the curve of an ellipse is given and the candidate is required to complete the ellipse. Of course the first thing he has to do is to find the transverse and conjugate diameters, by means of Prob. XCVIII., but it may so happen that the arc is not large enough for the line E F, which passes through the centre, to touch it in two places. How then is the centre to be found? In this manner. Draw another pair of parallel chords and bisect them, joining the points of bisection and producing the line till it meets the former line E F, which will be the centre at the point G. Having found the centre, the axes are obtained as in Prob. XCVIII., and the ellipse may be developed as in Prob. XCV. or XCVI.

PROBLEM C.

To draw a tangent *to the curve of an ellipse at a given point of contact* A.

Draw the transverse diameter B C and find the foci D and E (Problem XCVI.) From these points draw lines D A and E A and produce D A to F. Bisect the external angle E A F by the line C A; then

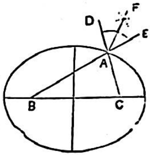


C A produced is the tangent required.

PROBLEM CI.

To draw a perpendicular *to the curve of an* ellipse *from a given point* A.

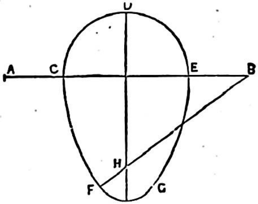
Draw the transverse axis, and find the foci B and C (P. XCVI.) Draw lines B A, C A, and produce them making the angle D A E. Bisect this angle by line A F; then



A F is the perpendicular required.

PROBLEM CII.

To draw an oval *by arcs of circles.*

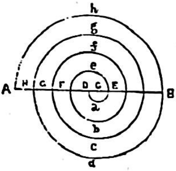


Upon any straight line A B describe a semicircle C D E, equal in diameter to the oval required. From C and E, with the radius of the semicircle, mark off A and B. From A and B, with radius A E, describe the arcs E G and C F. From A or B draw a straight line cutting the transverse diameter in H, and touching the opposite arc in G or F. From centre H, with radius H F, describe the arc F G, which completes the oval.

Note.—The oval may be made longer or shorter by increasing or diminishing the transverse diameter.

PROBLEM CIII.

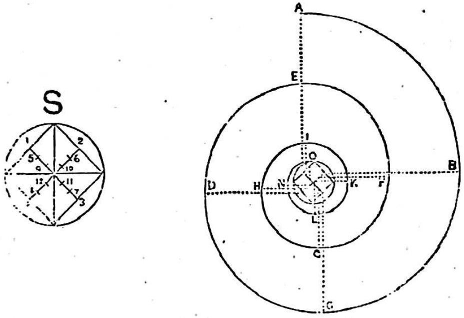
To construct a common Spiral ; A B *being the diameter.*



Take C as the eye of the spiral; from centre C with any radius describe the semicirle D *a* E. From centre D with D E as radius describe semicircle *e*, cutting A B in F. Again from C as centre, with C F as radius, describe the semicircle *b*, and so on the spiral may consist of any number of semicircles described alternately from C and D as centres.

PROBLEM CIV.

To construct the Spiral, *known as the* Ionic Volute.

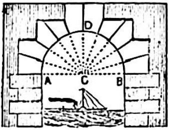


Let O be the centre of the diameter of the Volute. On one-fourth of the radius from O describe a circle which is called the eye of the Volute. Inscribe a square in this circle whose diameters are vertical. Divide each of these diameters into six equal parts, and number the divisions as marked in S, a larger representation of the eye. The Volute is now to be constructed by twelve consecutive arcs, of which these twelve points in their numerical order are the consecutive centres. The first arc, A B, is described from the centre 1 to the line 1 2 produced. The second arc, B C, from the centre 2 to the line 2 3 produced. The third arc, from the centre 3 to the line 3 4 produced, and so on describe arcs from each of the consecutive centres until the Volute is completed. (See the Ionic column on page 66.)

REMARKS ON APPLICATION.

To give the pupil an interest in Practical Geometry, the teacher should constantly bring before his notice the different uses to which the various constructions can be applied. These are so numerous that we cannot admire the furniture that ornaments our rooms, or the buildings that decorate our streets, without detecting many familiar applications. The following are a few illustrations :—

To find the Joints *of the* Arch Stones *of a Circular Arch having a given span* A B.



FIRST.—**If the arch be a semicircle.**

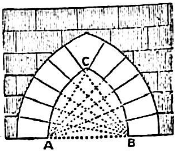
On A B construct the semicircle A D B, and divide it into as many equal parts as the number of arch stones required. From C, the centre of the arc A D B, draw radii passing through these divisions; these will give the direction of the joints as shown in the illustration.

SECOND.—**If the perpendicular height of the arc be given.**

Bisect the span A B, and place the perpendicular height C D at right angles to it. Through the three points A, D, and B, draw an arc. Divide this arc as the number of stones require, and from the centre of the arc draw radii through the divisions, which will give the position of the joints of the stones.

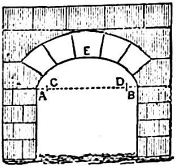
To find the **Joints** *of the* **Arch Stones** *of a Gothic Arch having a given span* A B.

From A and B as centres, with A B as radius describe arcs cutting in C. Divide these arcs similarly as the number of stones require. From A and B draw lines through these divisions which will mark the joints of the arch stones required.



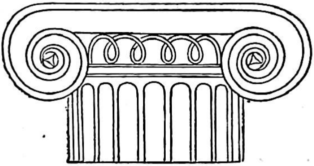
To find the **Joints** *of the* **Arch Stones** *of an Elliptical Arch having a given span* A B.

Construct the semi-ellipse A E B, and divide the curve as the number of stones require Find the focii C and D. and at each of the divisions erect perpendiculars as shown in the illustration. These perpendiculars mark the joints of the arch stones required.



THE IONIC VOLUTE.

The following figure illustrates the application of the **Ionic Volute** in ornamenting the Ionic pillar. (See Problem CIV.)



QUESTIONS AND EXERCISES.

1.—Construct an ellipse in three different ways, the transverse and conjugate diameters A B and C D being given.

2.—Complete the ellipse of which any curve A B is an exact quarter. (*See Problem XCIX.*)

3.—Draw a tangent touching the curve of an ellipse at any given point A, and also a line perpendicular to the curve, from that point.

4.—Draw a rectangle having sides of 3″ and 2½″, and in it inscribe an ellipse.

5.—Draw the plan of an elliptical arch built of nine stones of equal base. The arch has for its span 4″, and for its vertical height 1¾″.

SECTION X.

SIMILAR FIGURES.

Similar figures have their angles *equal*, and their corresponding sides *proportional*.

All Equilateral Triangles, Squares, and **Regular Polygons** of the same name, are similar. Other Quadrilateral Figures, Triangles, and Polygons can be constructed similar to given ones, by making their angles equal.

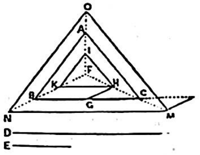
The following are general methods for inscribing and describing Similar Figures.

PROBLEM CV.

To inscribe within and equidistant from the sides of a given triangle A B C, *a* **similar triangle**, *one of whose sides is equal to a given line* E.

Bisect the angles of the triangles A B C by lines meeting in F. Make B G equal to E. Through G draw the line G H parallel to B F. Through H draw lines parallel to the sides of the given triangle cutting the bisecting lines in I and K. Join I K; then

I K H is a similar triangle inscribed within A B C.



PROBLEM CVI.

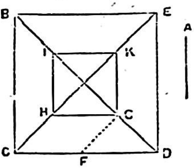
To describe about and equidistant from the sides of a given triangle A B C, *a* similar triangle, *one of whose sides is equal to a given line* D. (See figure CV.)

Produce the side B C to L, making B L equal to the line D. Through L draw L M parallel to the bisecting line B F, cutting F C produced in M. Through M draw the line M N parallel to the side B C, cutting F B produced in N. Through M and N draw the lines M O and N O parallel to the sides of the given triangle A B C, meeting F A produced in O; then

N O M is a similar triangle to A B C, described about it.

PROBLEM CVII.

To construct within a given square B C D E, *another* square *concentric with it, and having its side equal to a given line* A.



Bisect the angles of the square by the diagonals B D and C E. On C D cut off C F equal to A. Through F draw F G parallel to C E, and through G draw G H parallel to C D. Through H and G draw H I and G K parallel to B C. Join I K; then

I H G K is the concentric square required.

It will be noticed that this method is only a repetition of that used in Problem CV for inscribing a similar triangle, and if a square is required to be described about a given square, make C D produced equal to the required side, and proceed just as in Problem CVI.

PROBLEM CVIII.

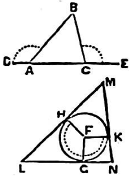
To inscribe a similar hexagon having a side equal to a given line A B within any given hexagon.

The methods are exactly similar to those employed in Problem CVII.

Any similar Polygon, having a given side, can be inscribed within, or described about a given regular polygon in the same manner.

PROBLEM CIX.

To describe a **triangle** *about a given circle, similar to a given triangle* A B C.

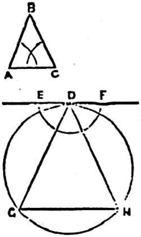


Produce one side of the given triangle A C each way to D and E. Find the centre F of the given circle H G K (Prob. XLIII) and draw any radius F G. Draw another radius F H making with F G an angle H F G equal to the angle B A D. Draw also another radius F K, making with F G an angle G F K equal to the angle B C E. Draw tangents at H, G, K, meeting in L, M, and N (P. XLVII.): then

L M N is the similar triangle required, described about the given circle H G K.

PROBLEM CX.

To inscribe a **triangle** *in a circle, similar to a given triangle* A B C.



Draw a tangent E D F at any point D in the circumference (P. XLVII). From D draw the line D G, making with E D an angle equal to the given angle B A C, and cutting the circumference in G. From D draw also the line D H, making with F D an angle equal to the given angle A C B, and cutting the circumference in H. Join G H; then

D G H is the similar triangle required inscribed in the given circle.

PROBLEM CXI.

To cut off a segment from a given circle which shall contain an angle equal to any given angle A. (See last Problem.)

Any angle contained in the segment D H G, is equal to the angle E D G and therefore to the given angle A.

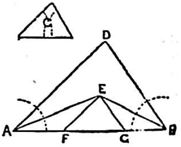
Note.—All angles in the same segment are equal to each other, and therefore any angle drawn in the segment D H G will be equal to the angle D H G.

PROBLEM CXII.

To construct a triangle similar to a given triangle C, *and having its* Perimeter *equal to a given straight line* A B.

Upon A B construct a triangle A D B, whose angles are equal to those of the given triangle C. Bisect the angles A and B by lines meeting in E. Through E draw lines E F and E G parallel to the sides D A and D B, meeting A B in F and G; then

F E G is the required similar triangle, having its perimeter, or three sides, equal to the given line A B.



QUESTIONS AND EXERCISES.

1.—Within an Octagon whose base is one inch, inscribe a similar concentric Octagon whose base is half an inch.

2.—About a square garden whose side is two furlongs, there is a promenade whose breadth is half a furlong, their boundaries are similar—draw a plan representing them. (*Scale two inches to a furlong.*)

3.—About a given Heptagon whose base is one inch, describe a similar Heptagon whose base is two inches.

4.—The base of a scalene triangle is ½″, its angles are respectively 40°, 60° and 80°, describe a similar triangle the base of which is ¾″.

5.—In any given circle inscribe a triangle having angles of 20°, 70° and 90°, respectively.

6.—About a given circle describe a triangle having angles of 100°, 60° and 20°, respectively.

7.—From a circle, whose radius is 1¾″, cut off a segment which shall contain an angle of 50°. (*See Problem CXI.*)

SECTION XI.

PROPORTIONAL LINES.

ILLUSTRATION.—*2 is to 4, so is* 6 *to* 12.

In this illustration the *relationship* or *ratio* that exists between the first two numbers is the same as that which exists between the last two, 2 being one half of 4, and 6 one half of 12; therefore, these four numbers are said to be in **Proportion** and the last term is called the

Fourth Proportional.

The *first* and *fourth* terms of a Proportion are called **Extremes**; as 2 and 12.

The *second* and *third* terms of a Proportion are called **Means**; as 4 and 6.

The product of the **Extremes** equals the product of the **Means**, e.g.— $2 \times 12 = 4 \times 6$.

Consequently if the product of the means be divided by the first extreme we the other extreme or fourth proportional; e.g.

$$\frac{4 \times 6}{2} = 12 = \textit{the fourth proportional.}$$

ILLUSTRATION.—2 : 6 : : 6 : 18.

When the two means are the same number, as in this illustration, the last term as 18 is called the

Third Proportional,

that is, of the three numbers employed, the third number is as much greater or less than the second as the second is greater or less than the first.

The *middle term*, 6, of these three numbers is called the

Mean Proportional.

The *third proportional* is found by dividing the square of the second or mean proportional by the first; e.g.

$$\frac{6^2}{2} \text{ or } \frac{6 \times 6}{2} = 18 = \textit{the third proportional.}$$

The mean proportional between any two numbers is found by extracting the square root of their product; e.g.

$2 \times 18 = 36$; the square root of $36 = 6 =$ *the mean proportional.*

Let us now apply these truths to Geometry and see how lines and surfaces are proportionally divided.

PROBLEM CXIII.

To find a **fourth proportional** *to three given lines,* A, B, *and* C

1st.—*A fourth proportional less.*

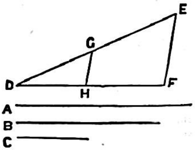
Make line D E equal A, and from D draw line D F at any angle, equal to B. Join F E. From D E cut off D G equal to C. Through G draw G H parallel to E F, cutting D F in H; then

D H *is the fourth proportional less, i.e.,*

D E : D F : : D G : D H, or

A : B : : C : D H. or

If A = 12 ft., B 8 ft., and C 6 ft., then D E = 4 ft.



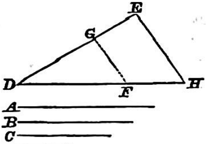
PROBLEM CXIV.

2nd.—*A fourth proportional greater.*

Draw D G equal to C, and D F at any angle equal to B. Join F G and produce D G to E, making D E equal to A. Through E draw E H parallel to G F, to meet D F produced in H, then

D H *is the fourth proportional greater, i.e.,*

D G : D F : : D E : D H.



PROBLEM CXV.

To find a **third proportional** *between two given lines* A *and* B.

1st.—*A third proportional less.*

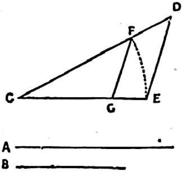
Make line C D equal to A, and from C draw C E, at any angle equal to B. Join D E. From C cut off C F equal to C E. Through F draw line F G parallel to D E cutting C E in G; then

C G *is the third proportional required, i.e.,*

C D : C E : : C E or C F : C G, or

A : B : : B : C G; or

If A = 9 ft., and B 6 ft., then C G = 4 ft.



PROBLEM CXVI.

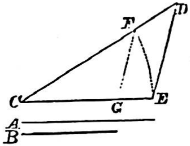
2nd.—*A third proportional greater.*

Draw lines C G and C F at any angle equal to B and A. Join F G. From centre C, with F C as radius, describe an arc F E cutting C G produced in E. Draw E D parallel to G F meeting C F produced in D; then

CD *is the third proportional greater, i.e.,*

C G : C F : : C F : C D, or

B : A : : A : C D.

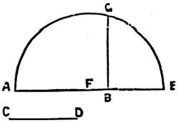


Note.—The pupil will remember this process better if he notices that to get the third proportional less, the radius of the arc E F is the shorter of the two given lines, but to get the third proportional greater, it is the longer of the two given lines.

PROBLEM CXVII.

To find the mean proportional *between two given lines* A B *and* C D.

Produce A B to E. Make B E equal to C D. Bisect A E in F and describe the semicircle A G E. Erect B G perpendicular to A E cutting the semicircle in G. Then B G is the mean or middle proportional between A B and C D, that is if A B=9 ft. and C D=4 ft., then B G= 6ft.



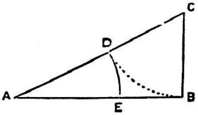
PROBLEM CXVIII.

To divide a straight line A B *into* extreme *and* mean *proportion.*

From B erect perpendicular B C equal in length to half of A B. Join A C. From C with radius C B describe an arc cutting A C in D. From A with radius A D describe an arc cutting A B in E; then

B E *is the extreme, and* A E *the mean proportion of* A B, *i.e.,*

E B : A E : : A E to A B.

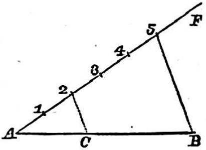


PROBLEM CXIX.

To divide any straight line A B *in the point* C, *so that* A C : C B : : 2 : 3.

At A make the line A F of indefinite length, and at any angle. On A F mark off any 2 equal distances and from A 2 mark off three similar distances to 5. Join 5 B. Through 2 draw 2 C parallel to 5 B. Then A B is divided into two parts, A C and C B, so that

A C : C B : : 2 : 3.



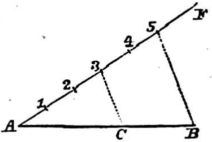
PROBLEM CXX.

To divide any straight line A B *in the point* C, *so that the whole* A B *is to one part* A C *as* 5 : 3.

At A make the line A F of indefinite length, and at any angle to A B. Take any 5 equal distances on A F and join 5 B. At the point 3 draw a line 3 C parallel to 5 B. Then A B is divided in C, so that

A B : A C : : 5 : 3.

Note.—A B is also divided, so that
A C : C B : : 3 : 2, also
A B : C B : : 5 : 2.

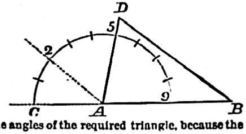


PROBLEM CXXI.

To construct a triangle *on a straight line* A B, *so that the three angles are in the proportion of* 2 : 3 : 4.

From A, with any radius, describe an arc and divide it into 9 (2 + 3 + 4 = 9) equal parts. Join 2 A and 5 A. Then the three angles, 2 A C, 5 A 2, and 5 A 9 are in the proportion of 2 : 3 : 4 They are also equal to two right angles, and are therefore the angles of the required triangle, because the three angles of any triangle are together equal to two right angles. Through B draw the line B D parallel to A 2 until it meets A 5 produced in D. Then A B D is the required triangle, that is, the angles

A B D : B D A : D A B : : 2 : 3 : 4.



QUESTIONS AND EXERCISES.

1.—Draw three lines 1½″, 2″, 2¾″ in length respectively, and find their fourth proportional.

2.—Divide a line A B 3½″ long, into its mean and extreme proportion.

3.—Produce a line A B, 2″ long to point C, so that A B : B C : : 2 : 5.

4.—Find the length of A B the mean proportional between two lines 1½″ and 3¼″ long.

5.—The perimeter of a triangle is 3″, construct it so that its sides are in the proportion of 2, 3, and 4.

6.—Construct a triangle on base A B 2¼″ long, so that the angles are in the proportion of 1, 2, and 3.

SECTION XII.

EQUIVALENT AREAS.

The pupil should carefully study the following explanations and illustrations, in order to understand the relation in area between the various kinds of *parallelograms* and *triangles*.

Fig. 1.

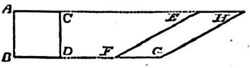
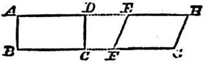


Fig. 2.



First.—**Parallelograms standing on the same or equal bases, and between the same parallels, are equal to one another.**

Therefore by making their bases and altitudes equal the pupil can readily convert.

(*a*) A *square* into an equivalent *rhomboid*, and a rhomboid into an equivalent *square* or *oblong*.

In this illustration the square A B D C equals the rhomboid E F G H, because they stand upon equal bases, B D and F G, and between the same parallels B G and A H.

(*b*) A *rectangle* can be converted into an equivalent *rhomboid*, or a rhomboid into a rectangle.

In the last illustration Fig. 2, the rectangle A B C D equals the rhomboid E F G H, because they stand on equal bases, B C and F G, and between the same parallels, B G and A H.

SECOND.—**Triangles standing upon the same or equal bases, and between the same parallels, are equal to one another.**

Therefore by making their bases and altitudes equal, the pupil can convert a triangle of one shape into any equivalent triangle of another shape, e.g.

Fig. 1.

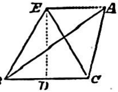
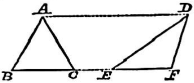


Fig. 2.



(*a*) A *scalene triangle* can be converted into an equivalent isosceles triangle.

In Fig. 1 the isosceles triangle E B C is equal to the scalene triangle A B C.

(*b*) An *equilateral triangle* can be converted into an equivalent scalene triangle.

In Fig. 2 the scalene triangle D E F is equal to the equilateral triangle A B C because they stand upon equal bases and between the same parallels.

(*c*) An *acute angled triangle* can be converted into an equivalent right angled triangle.

THIRD.—**If a parallelogram and a triangle stand upon the same or equal bases, and between the same parallels, the triangle is half the parallelogram ;** also

A triangle standing upon double the base of a parallelogram, and having the same vertical height, is equal to the parallelogram ; also

A triangle standing upon the same or equal base as a parallelogram, and having double its vertical height, is equal to the parallelogram. Therefore—

Fig. 1.

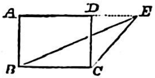
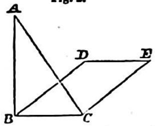


Fig. 2.



(*a*) A *parallelogram* can be drawn double a triangle, and a triangle half a parallelogram.

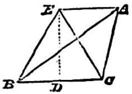
In Fig. 1 the parallelogram A B C D is double the triangle E B C.

(*b*) A *triangle* can be drawn equal to a parallelogram, and a parallelogram equal to a triangle.

In Fig. 2 the triangle A B C is equal to the parallelogram D B C E, because it stands upon the same base, but has double the vertical height.

PROBLEM CXXII.

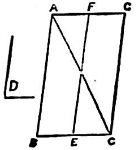
To construct an isosceles triangle *on* B C *equal in area to the given triangle* A B C.



As we have seen before, the vertical angle of an isosceles triangle stands exactly over the centre of the base; therefore bisect B C in D, and erect a perpendicular D E of the same vertical height as that of the triangle A B C. Join E B and E C. Then the isosceles triangle B E C = the given triangle A B C.

PROBLEM CXXIII.

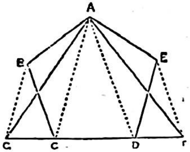
To construct a parallelogram *equal in area to a given triangle* A B C, *and having one of its angles equal to a given angle* D.



Bisect the base B C in E. At E make the angle C E F equal to the given angle D Through A draw A G parallel to B C cutting E F in F, and through C draw C G parallel to E F cutting A G in G. Then the parallelogram F E C G is equal to the given triangle A B C., and it has its angle F E C equal to the given angle D.

PROBLEM CXXIV.

To construct a quadrilateral *figure equal in area to any pentagon* A B C D E, *or a* triangle *equal in area to a quadrilateral figure or pentagon.*

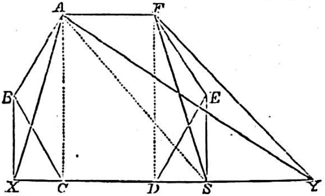


Join any two alternate angles A and D. Through E draw E F parallel to A D meeting C D produced in F. Join A F. Then the triangle A F D is equal to the triangle A E D because it is on the same base A D and between the same parallels A D and E F, and therefore the quadrilateral A B C F is equal to the pentagon A B C D E. In the same manner make the triangle A G C equal to the triangle A B C. Then the whole triangle A G F is both equal to the quadrilateral A B C F and to the given polygon A B C D E.

Note.—Any polygon can thus be reduced step by step to any figure of the same area having a less number of sides. For the sake of further illustrating this method the next problem is given.

PROBLEM CXXV.

To reduce any rectilineal figure *to an equivalent figure having a less number of sides. (Say a hexagon to a triangle.)*



1st.—Join F D, and through E draw E S parallel to F D, cutting C D produced in S. Join F S. Then the triangle F S D is equal to the triangle F E D, and therefore the five sided figure A B C S F is equal to the hexagon A B C D E F.

2nd.—Join A C, and through B draw B X parallel A C, meeting C D produced in X. Then the triangle A B C is equal to the triangle A X C, and therefore the quadrilateral figure A X S F is equal to the figure A B C S F.

3rd.—Join A S, and through F draw F Y parallel to A S, meeting X S produced in Y. Then the triangle A Y S equals the triangle A F S, and therefore the triangle A X Y equals the quadrilateral figure A X S F, or the pentagonal figure A B C S F, or the given hexagon A B C D E F.

QUESTIONS AND EXERCISES.

1. Draw an equilateral triangle having a base of 3″, and construct a rectangle equal to it in area.

2. Construct a rhombus having a base of 3½″ and two angles of 45°, and make a triangle of equal area having one angle of 70°.

3. Make a right angled triangle on a base of 2½ inches, and having a height of 1¾ inches; and construct an isosceles triangle of equal area.

4. Construct a square having a base of 1½″ and make a rhomboid of equal area, two of its angles being each 75°.

5. Draw a rhomboid on a base of 1¼″, and construct an isosceles triangle of equal area.

6. Construct a square having a base of 3″, and also a scalene triangle of equal area, one of the angles at the base being 30°.

SECTION XIII.

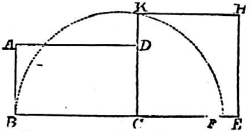
PROPORTIONAL & EQUIVALENT AREAS.

The Problems in this Section are based upon the principles of construction contained in Sections XI. and XII.

PROBLEM CXXVI.

To construct a square *equal in area to a given rectangle* A B C D.

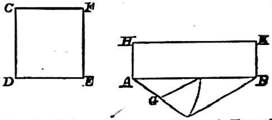
Produce B C to F making C F equal to C D. Find C K the mean proportional to the two sides B C and C F, which is a side of the required square. On C K describe the square C K H E. Then



the square C K H E = the rectangle A B C D.

PROBLEM CXXVII.

To construct a rectangle *on a given base* A B *equal in area to a given square* C D E F.



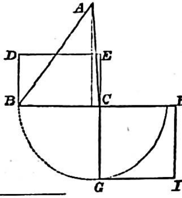
This is the converse of the last problem, and it is quite evident that the side required to complete the rectangle on A B will be the third proportional to the side A B and D E.

Find A G, this third proportional. At A erect the perpendicular A H equal to A G and complete the rectangle. Then

the rectangle A H K B = the square C D E F.

PROBLEM CXXVIII.

To construct a square *equal in area to a given triangle* A B C.



1st. Convert the triangle into an equivalent rectangle D B C E.

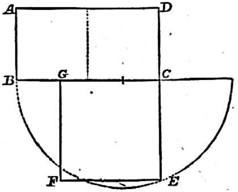
2nd. Convert this rectangle D B C E into an equivalent square C G I H. (Prob CXXVI.) Then

the square C G I H = the triangle A B C.

PROBLEM CXXIX.

To construct a square *that shall have an area of two square inches (or any other number of square inches.)*

N.B.—This figure, as well as the next, is cut on a scale of half-an-inch to the inch.



1st. Construct a rectangle A B C D of two square inches, viz.:—a rectangle having two inches for one side B C, and one inch for the other side A B.

2nd. Construct the square C E F G equal to the rectangle A B C D (Prob. CXXVI.) Then

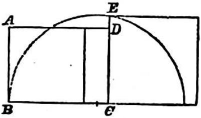
the square C E F G = 2 square inches.

PROBLEM CXXX.

A **square** *is* 1¼ *square inches in area, determine the length of its side.*

1st. Construct a rectangle A B C D containing 1¼ square inches, viz.:—a rectangle having one side 1¼ inches long and the other side 1 inch long.

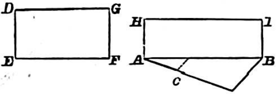
2nd. Find C E the mean proportional to B C and C D, which is the side of a square containing 1¼ square inches.



PROBLEM CXXXI.

Upon a given base A B *to construct a* **parallelogram** *equal in area to a given parallelogram* D E F G.

The height of the required parallelogram will be the fourth proportional to A B, E F, and E D. Find A C this fourth proportional. At A erect the



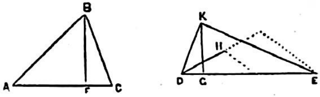
perpendicular A H equal to A C and complete the parallelogram H A B I. Then the parallelogram H A B I = the parallelogram D E F G.

Note.—A parallelogram is equal in area to a triangle of equal base and twice the altitude.

Therefore, a *parallelogram* can be constructed on any line equal in area to a given *triangle*, by first making a parallelogram on the base of the given triangle equal to that triangle, and then making a parallelogram on the given base equal to the parallelogram obtained.

PROBLEM CXXXII.

To construct a **triangle** *on a given base D E which will be equal in area to a given triangle A B C.*



Let A B C be the given triangle, and D E the base of the required triangle. Let fall upon A C the perpendicular B F. The fourth proportional to the three

lines D E, A C, and B F will be the perpendicular height of the required triangle. Find D H this fourth proportional, (Prob. CXIII) and erect it perpendicular to D E, as K G, from any point in the base, and construct the triangle D K E which will be equal in area to the given triangle A B C.

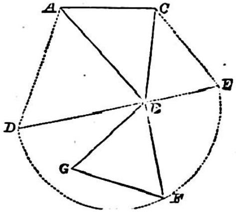
NOTE.—*If the perpendicular height K G had been given instead of the base D E, it is quite evident that D E must have been the fourth proportional greater.*

Note.—D E : A C : : B F : K G.

Compare this Problem with Problem CXXXI.

PROBLEM CXXXIII.

To construct an equilateral triangle *equal in area to a given triangle* A B C.



On one of the sides of the given triangle as A B, construct an equilateral triangle A D B. Produce D B, one of the sides of this equilateral triangle, and through C, the apex of the given triangle, draw a line C E parallel to the base A B, meeting D B produced in E. Then the mean proportional B F, between D B and B E, is the base of the required equilateral triangle. Construct it. Then

the equilateral triangle B G F = the triangle A B C.

SPECIAL EXERCISES.

1. On a given base A B, 1" long, construct a parallelogram equal in area to a square, the base of which is $\frac{1}{2}$ an inch long.

2. On a given base A B, $\frac{3}{4}$ of an inch long, construct a parallelogram equal in area to a trapezium, the four sides of which are 1", $1\frac{1}{4}$", 2" and $2\frac{1}{4}$" respectively.

3. To construct an equilateral triangle equal in area to a square, the base of which is 1".

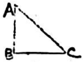
4. On a given base A B, $1\frac{3}{8}$" long, construct a parallelogram equal in area to a hexagon, the sides of which are $\frac{1}{2}$ an inch long.

5. Complete the rectangle on base, A B 2" long, equal in area to a square, having a side of $1\frac{3}{8}$".

SECTION XIV.

THE RIGHT ANGLED TRIANGLE.

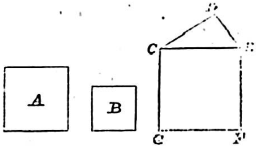
In every right angled triangle the square of the side opposite the right angle is equal to the squares of the sides containing the right angle. (Euclid, 47 Prob., 1st Book.)



In the adjoining triangle the square described on A C is equal in area to the squares described upon A B and B C., that is $AC^2 = AB^2 + BC^2$.

PROBLEM CXXXIV.

Construct a **square** *equal in area to two given squares* A *and* B.



Make the bases of these squares the sides, C D and D E containing the right angle C D E. Join C E. Then the square described on C E is equal to the squares described on C D and D E, or to the two squares A and B. On C E construct a square C G F E. Then the square C G F E = the squares A and B.

Note.—In the same manner we may construct a square equal in area to any number of squares, by first constructing a square equal in area to two of the squares, and then constructing a square equal in area to the one so found, and the next square, and so on for any number of squares.

PROBLEM CXXXV.

To make a **square** *equal in area to the difference between the areas of any two squares* A *and* B.

This Problem depends on the principle just explained. In Problem CXXXIV we found that $CE^2 = CD^2 + DE^2$, therefore $CD^2 = CE^2 + DE^2$, and also $DE^2 = CE^2 + CD^2$; hence the pupil can work this Exercise, and Nos. 4, 5, and 6, given in the Exercises on page 88.

(See illustration Prob. CXXXIV.)

NOTE.—The areas of similar shaped figures are in the same proportion as the squares on their similar sides. (See Euclid Book VI.) Hence

1st.—If the lines C D and D E represent the *bases of two similar triangles* the line C E represents the base of a similar triangle, equal in area to the areas of the other two.

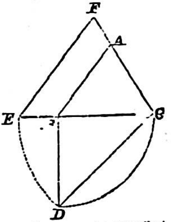
2nd.—If the lines C D and D E represent the diameters or radii of *two circles*, then the line C E represents the diameter or radius of a circle equal in area to the areas of the other two.

3rd.—If the lines C D and D E represent the diameters or radii of circles containing *two similar polygons*, or the bases of two similar polygons, then the line C E represents the diameter or radius of a circle containing a similar polygon, or the base of a similar polygon, equal in area to the areas of the other two.

Application to Triangles, Circles, and Polygons.

PROBLEM CXXXVI.

To construct a triangle *similar to the triangle* A B C *but of twice its area.* (Ratio 2 : 1.)



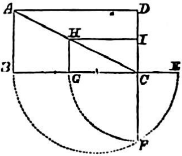
Let fall B D perpendicular to B C and equal to it. Join D C. Then the square on D C is equal to the squares on B C and B D, that is double the square on B C. Produce C B to E making C E equal to C D, then the square on C E would be double the square on B C. From E draw E F parallel to B A to meet C A produced in F. Then the triangle E F C is similar to A B C but double its area.

Note.—In the same manner a *quadrangle, circle,* or *polygon* can be described equal to two, three, or any other number of times a given quadrangle, circle, or polygon, which the pupil should now construct for himself.

PROBLEM CXXXVII.

To construct a quadrilateral, or any other figure, the area of which shall be one-third (*or any other proportion*) of a similar figure.

1st.—*To construct a* parallelogram *having one-third of the area of a given parallelogram,* A B C D. (Ratio 1 : 3)



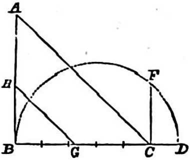
1. Divide B C into 3 equal parts and produce it to E, making C E equal to one of these parts.
2. Find C F the mean proportional between B C and C E. Then C F is the length of the base of the parallelogram required.
3. From C mark off C G equal to C F. Join A C and draw G H and H I parallels to C D and B C; then

the parallelogram I C G H = $\frac{1}{3}$ A B C D.

Note.—If concentric centres are required find C F and then apply the principles of construction employed in Prob. CVII.

PROBLEM CXXXVIII.

2nd.—*To construct a* triangle *one-fifth the area of a given triangle* A B C.

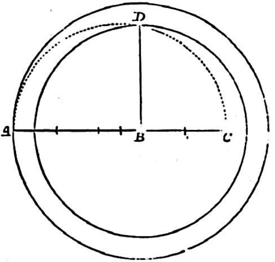


Produce B C to D making C D equal to $\frac{1}{4}$ of B C. Find C F the mean proportional to B C and C D, which is the length of the base of the required triangle. Make B G equal to C F, and from G draw G H parallel to A C. Then the area of the triangle H B G is one-fifth of the area of A B C: *i.e.*

H B G = $\frac{1}{5}$ A B C

PROBLEM CXXXIX.

3rd.—*To construct a* circle *two-thirds the area of a given circle.*

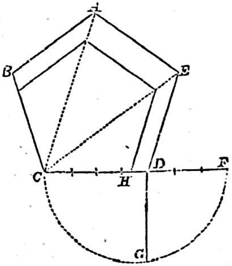


Draw the radius A B and divide it into three equal parts. Produce it to C, making B C equal to two of these divisions. Find B D the mean proportional between A B and B C, which is the radius of the required circle. From centre B, with distance B D, describe the inner circle, the area enclosed by which will be *two-thirds* that of the area enclosed by the outer circle.

PROBLEM CXL.

4th.—*To construct a* polygon *equal in area to three-fourths of that of a given polygon, (in this case a pentagon)* A B C D E.

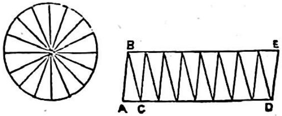
Divide the base C D into four equal parts and produce it to F, making D F equal to three of these parts. Find D G the mean proportional to C D and D F. Then D G is the base of the required figure. From C mark off C H equal to D G. Join A C and E C and from H draw a series of parallels, as shown in the diagram to complete the required figure. Then the smaller Pentagon equals ¾ of the larger Pentagon A B C D E.



Note.—The pupil may be asked to construct a *square, triangle, polygon* or *circle*, any number of times greater than a given square, triangle, polygon or circle. If so the required base or diameter may be found by producing the base of the given square, triangle or polygon, or the diameter of the given circle, *two, three*, or the required number of times greater, and then finding as in the last 4 problems the mean proportion. *(See also Problem CXXXVI and its note.)*

THE CIRCLE AND ITS EQUIVALENT TRIANGLES AND QUADRILATERALS.

The area of a circle is equal to that of a rectangle whose longest sides are equal to half the circumference, and shortest sides to the radius of the circle.



Proof.—Divide the given circle, as in the above illustration, into any number of equal triangles, say 16. Construct a triangle A B C equal to one of these triangles, and produce the side A C to D, making the distance A D equal to eight times A C, that is to one half of the circumference. Complete the parallelogram A B E D, of which A D and A B are two adjacent sides. Construct on A D eight consecutive triangles equal to A B C and having their sides parallel to A B. Then the whole figure A B E D consists of sixteen triangles, each equal to A B C, which is the sixteenth part of the whole circle; therefore the parallelogram A B E D is approximately equal to the given circle.*

Therefore any circle can be converted into a *rectangle, square, rhomboid,* or *rhombus* of the same area.

Explanation.—Any triangle having A D for its base, and twice the altitude of the parallelogram A B E D for its height, would be equal to the parallelogram A B E D, and therefore to the given circle; but A D is half the circumference,* and as the altitude of the parallelogram is by construction equal to the radius of the circle, therefore twice this altitude is equal to the diameter.

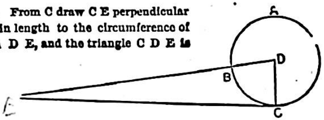
Therefore, any circle can be converted into a triangle of the same area, by taking *half its circumference for its base and diameter for its altitude;* or what amounts to the same thing, *its whole circumference for base and radius for altitude.*

APPLICATION.

PROBLEM CXLI.

To construct a triangle *equal in area to any given circle.*

Draw any radius D C. From C draw C E perpendicular to D C and equal in length to the circumference of the circle.* Join D E, and the triangle C D E is equal in area to the given circle A B C.



Also do this Problem by taking half the circumference of the circle for its base and diameter for altitude.*

* Note.—Theoretically the circumference of a circle cannot be exactly represented by a line, nor its area by a rectilinear figure of any kind; but practically the difference is so slight we need not notice it. The circumference of any circle is about $3\frac{1}{7}$ times the diameter.

QUESTIONS AND EXERCISES.

1. Construct a triangle equal in area to two similar triangles, the bases of which are respectively $1''$ and $1\frac{5}{8}''$.

2. Construct a circle, the area of which shall be equal to two circles, the radius of each being $\frac{1}{2}''$ and $\frac{5}{8}''$ respectively.

3. Construct a regular hexagon equal in area to two given regular hexagons, the bases of which are respectively $1''$ and $1\frac{7}{8}''$.

4. Construct a triangle equal in area to the difference between two similar triangles, the bases of which are respectively $2''$ and $1\frac{1}{3}''$.

5. Construct a circle equal in area to the difference between two other circles, the diameters of which are respectively $2\frac{1}{4}''$ and $1\frac{5}{8}''$.

6. Construct a hexagon equal in area to the difference between two hexagons, the diameters of which are respectively $\frac{1}{2}''$ and $2\frac{1}{2}''$.

7. Construct a triangle on a base $\frac{7}{8}''$ similar to a given equilateral triangle on a base of $1\frac{5}{8}''$.

8. Describe a circle having a radius of $1\frac{1}{8}''$, and construct a *rhomboid* having an angle at the base of 45° equal to it in area.

9. A circle has a circumference of $6''$, construct a square equal to it in area. (N.B.—Construct a rectangle first.)

10. On base A B, $1''$ long, construct a right-angled triangle equal in area to a circle of $2''$ diameter. (N.B.—1st. Make a right-angled triangle, having $\frac{1}{2}$ circumference for base, and diameter for altitude. 2nd. Construct on A B a triangle of equal area.)

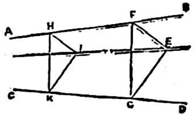
SECTION XV.

CONVERGENT LINES.

PROBLEM CXLII.

Through a given point E, *to draw a line which would if produced, pass through the angular point towards which the two given lines* A B *and* C D *converge.*

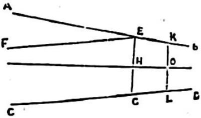
Draw any convenient line as F G. Join F E and G E. Draw any line H K parallel to F G. Through H and K draw lines H I and K I parallel to F E and G E cutting each other in I. Through E and I draw line E I, which produced is the convergent line required.



PROBLEM CXLIII.

To draw a line bisecting the angle between two converging lines A B *and* C D, *when the angular point is inaccessible.*

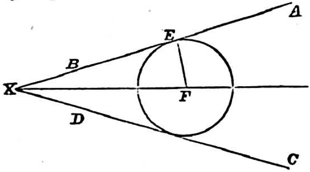
Through any point E in A B draw a line E F parallel to C D. Bisect the angle B E F by line E G. Through any point K between E and B, draw line K L parallel to E G. Bisect each of these lines in H and O. Join O H, which produced is the bisecting line required.



THE LINE AND CIRCLE AND TANGENTIAL ARCS.

PROBLEM CXLIV.

To draw a circle *(or an arc) which shall be tangential to any two converging lines,* A B *and* C D, *and which shall touch one of the lines in the given point* E.



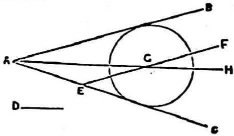
Draw the lines A B and C D to meet in X. Bisect the angle A X C by the line X F G. From point E draw a line E F perpendicular to A B, meeting X G in F. Then

E F is the radius of the required circle. Describe it.

Note.—If the lines A B and C D were inaccessible, as in Problem CXLIII, the bisecting line X F G should be obtained as shown in that Problem.

PROBLEM CXLV.

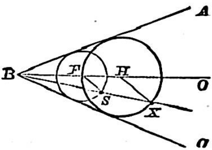
To draw a circle *with a given radius* D, *which shall be tangential to any two converging lines* A B *and* A C.



Bisect the angle B A C by the line A H. Draw E F parallel to A B at distance D, cutting A H in G. Then G is the centre of the required circle. *Describe it.*

PROBLEM CXLVI.

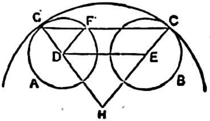
To inscribe a circle *in a given angle* A B C, *which shall pass through a given point* X.



Bisect the given angle A B C by line B O. Take any convenient point, F in B O, and with F as centre describe a circle touching the lines A B and C D. Join X B cutting the circle in S. Draw the radius S F, and through X draw X H parallel to S F; then H is the centre of the required circle, of which the radius is X H.

PROBLEM CXLVII.

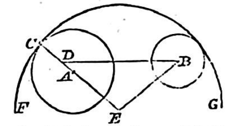
To describe a circle *or an* arc *tangential to and including two equal circles* A *and* B, *and touching one of them in a given point of contact* C.



Find the centres D and E of the given circles and join them. Draw line E C, and through D, the centre of the other circle, draw line D F parallel to E C. Join C F, and produce it, if necessary, to a point opposite C, as G. Join G D, and produce it to meet C E produced in H. Then H C is the radius of the required circle. Describe it.

PROBLEM CXLVIII.

To draw a circle (or an arc) *tangential to and including two unequal circles, and touching the largest or smallest at any given point* C.

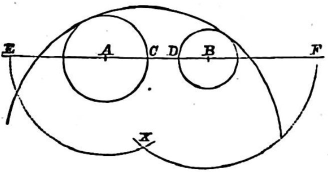


Find A and B the centres of the circle and join C A. Cut off from C A a part C D equal to the radius of the smaller circle. Join D B. Produce C D indefinitely. At B make the angle E B D equal to the angle E D B by the straight line B E, which meets C D produced in E. From centre E, with radius E C, describe the required arc F C G, which is tangential to the two circles and touches one of them in the given point C.

Note.—The principle is the same if the given point is in the small circle, the distance as C D in that case falling beyond C A the radius of the small circle.

PROBLEM CXLIX.

To draw an arc *(or circle) having a radius of one inch, which shall be tangential to two enequal circles* A *and* B *and include them.*

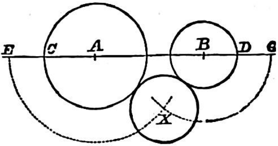


NOTE.—*The radius of the circle to include the other two, must be greater than the diameter of the greater circle.*

Join the centres A and B and produce the line indefinitely beyond the circumference of each circle. On the line A B make C E and D F, each equal to the radius of the required circle, viz.—one inch. From the centres A and B describe the intersecting arcs E X and F X. Then X is the centre of the required arc, which is tangential to the two circles and includes them both. *Describe the arc.*

PROBLEM CL.

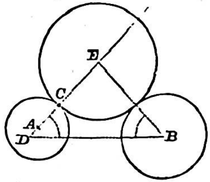
To draw a circle *having a radius of one quarter inch, tangential to the two unequal circles* A *and* B *externally.*



Join the centres A and B and produce the line indefinitely beyond the circumference of each circle. From C and D mark off C E and D G equal to the radius of the required circle, viz.:—1 quarter in. From A and B as centres, describe the intersecting arcs E X and G X. Then X, the point of intersection is the radius of the required circle, which is tangential to the circles A and B.

PROBLEM CLI.

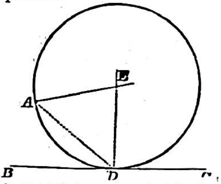
To draw a circle *(or an arc) externally tangential to two unequal circles, and touching one of them in a given point* C.



Find the centres A and B. Join A C and produce it indefinitely. Mark off, on C A produced, C D equal to the radius of the larger circle, and join B D. Draw line B E to meet D C produced in E, and making with D B, an angle equal to the angle B D E. Then E is the centre of the required circle. From centre E, with radius E C, describe the circle which is tangential to the other two and touches one of them in the point C.

PROBLEM CLII.

To construct a circle *which shall pass through any given point* A, *and touch a given line* B C *in a given point* D.

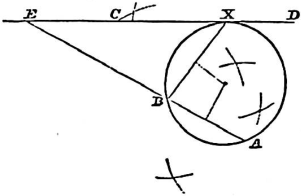


Draw D E at right angles to B C and join A D. At the point A make angle D A E equal to angle A D E. From the centre E, with distance E D, describe the circle required, which will pass through the point A and be tangential to B C at the point D.

Note.—The pupil should observe that the solution of this problem depends on the properties of the isosceles triangle; E A D being an isosceles triangle.

PROBLEM CLIII.

To construct a circle *which shall pass through the two given points* A *and* B, *and touch the given line* C D.

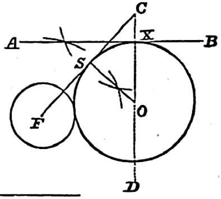


Join A B and produce it to meet D C produced in E. Find the mean proportional between the two lines A E and E B, and from E mark off E X equal in length to this mean proportional. Then X is the point at which the required circle drawn through A and B will touch the line C D. Through the three points A B X draw the circle.

PROBLEM CLIV.

To construct a circle *to touch the given line* A B *in the point* X, *and also the smaller circle.*

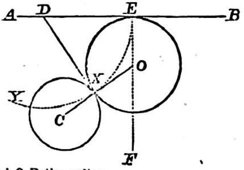
Through the point X, draw a perpendicular line C D of indefinite length. From X cut off X C equal in length to the radius of the given circle and join C F. Bisect C F in the point S and draw S O perpendicular to C F meeting C D in O. Then O is the centre of the required circle and O X the radius.



PROBLEM CLV.

To construct a circle *to touch a given circle in the point* X, *and the line* A B.

Find the centre C and draw line C X. Draw the tangent X D meeting the line A B in D. From D as centre, with radius X D, describe the arc E X Y meeting the line A B in E. From E draw E F perpendicular to A B. Produce C X to meet the perpendicular E F in O. Then O is the centre of the required circle and O E the radius.

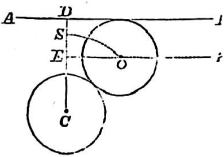


PROBLEM CLVI.

To construct a circle, *the radius of which is* $\frac{1}{4}''$ *to touch the given circle and the line* A B.

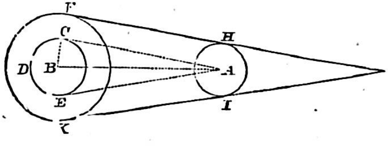
NOTE.—*The pupil has to find a point* $\frac{1}{4}''$ *from both the line and the circle.*

Find the centre C of the given circle and draw C D perpendicular to A B. From D mark off D E equal to the radius of the required circle, viz.:—$\frac{1}{4}$ inch, and draw the line E F parallel to A B. From centre C, with radius $\frac{1}{4}$ inch greater than the radius of the given circle, describe the arc S O meeting the line E F in O. Then O is the centre of the required circle which can now be described.



PROBLEM CLVII.

To draw one or two exterior tangents *common to two given circles.*

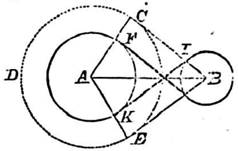


Join the centres A and B. From the centre B, with a distance equal to the difference between the radii of the two given circles, describe the inner circle C D E. From the centre A draw A C a tangent to the circle C D E. Join B C and produce it to cut the circle in F. Through the point F draw F H parallel to C A. Then F H is a tangent, and in the same manner I K is another tangent to the two circles.

PROBLEM CLVIII.

To draw one or two interior tangents *to two given circles.*

This is done by a similar process to the latter, only the radius which describes the circle C D E must be equal in length to the sum of the radii of both circles, instead of their difference as in the latter problem.



Note.—The pupil will readily notice the application of the principles involved in these two problems to various mechanical operations connected with wheels in machinery.

QUESTIONS AND EXERCISES.

1. Draw a circle 1″ in diameter and a line A B at any distance from it. Then draw a circle which shall touch both the given line and circle.

2. The diameters of two circles are 1½″ and 2″ respectively, draw a circle which shall touch the larger in a given point A.

3. Describe a circle which shall touch another in a given point, and also pass through any given point.

4. Draw a circle ½″ in radius which shall touch both lines of an angle of 40°.

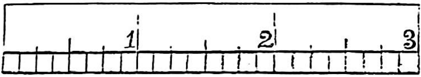
5. Any two lines A B and C D converge towards each other. Show how the angle at which they meet can be bisected when it is inaccessible.

6. Describe a circle having a radius of ¾″, and then describe two other circles of the same size which shall touch each other and the given circle in a point E Describe a circle which shall circumscribe the three circles

SECTION XVI.

SCALES.

DRAWINGS are usually made smaller than the objects they represent, but sometimes larger; for example, a drawing which represents the steeple of a church is made *smaller* than the steeple itself, whilst a drawing made to represent the mechanism of a watch is *larger* than the parts of the watch illustrated. Nevertheless, the various parts of the object represented, and which an artizan is required to copy in the work he has to do, must bear the same proportions to each other on paper as they do in the object itself; this is done by means of a **scale**. Thus we may draw a line *three inches long* as in the following figure and divide it into three equal parts. Then we may assume that each part stands for one yard, and so the whole line would stand for three times one yard, that is three yards.



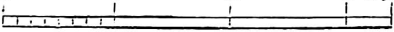
We may also divide each one of the three equal parts into eight equal divisions. Then each division might represent the *eighth part* of one yard, that is 4½ inches.

Note.—This scale is called a scale of $\frac{1}{36}$ because the line three inches is one-thirty-sixth part of the distance it represents, viz.: 3 yards.

PROBLEM CLIX.

Construct a plain scale to represent 27 yards. The scale being 8 yards to an inch.

0 8 16 24 27 yds.



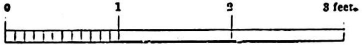
Since 1″ represents 8 yards it is quite evident 3″ will represent 24 yards. We must therefore first draw a line 3″ long to represent 24 yards. We have now to extend the line to represent 3 yards, but as one inch represents 8 yards we

divide the first inch into eight equal parts and each part represents 1 yard. Take now 3 of the equal parts so obtained and measure the line off from 24 to 27. Thus the whole line will represent 27 yards.

PROBLEM CLX.

Construct a scale of $\frac{1}{12}$ to represent feet and inches, making the scale long enough to represent 3 feet.

SCALE OF INCHES AND PARTS.



This means that the scale is to be considered 12 times less than the object it represents, which in this case is 3 feet. The scale must therefore measure $\frac{1}{12}$ of 3' which equals $\frac{36''}{12} = 3''$.

Draw a scale 3 inches long and this will represent 3 feet. Divide it into 3 parts and each part represents 1 foot, *i. e.* the scale is one inch to the foot. As the scale is to represent inches and there are 12" in a foot we must divide the first inch into 12 equal parts.

Note.—The same principle extends to the construction of scales of any size.

SPECIAL EXERCISES.

1. Construct the following scales to represent feet and inches :—

1.—A scale of $\frac{1}{32}$ to measure 16 feet.
„ $\frac{1}{16}$ „ 8 „
„ $\frac{1}{12}$ „ 10 „

2. Construct the following scales to represent yards and feet :—

2.—A scale of $\frac{1}{48}$ to measure 12 yards.
„ $\frac{1}{12}$ „ 3 „
„ $\frac{1}{18}$ „ 8 „

3. Construct a scale to represent 36 feet, on a scale of $\frac{1}{8}''$ to the foot, and then enlarge the scale to measure three times as much.

End of Plane Geometry.

SOLID GEOMETRY.

INTRODUCTION.

Solid Geometry treats of the representation of solid objects in two views upon a plane surface, for the convenience of mechanics, builders, and other artizans, who have to construct those objects from the drawings placed in their hands.

The *true proportions* and *size* of an object are represented in Solid Geometry, either on a larger or smaller scale, and thus it differs from Pictorial Perspective in which the object is represented as it appears to the eye.

Expl.—The *draughtsman* who draws the plan of any part of a machine for a mechanic to copy, and also the *architect* who draws the plan of a building for a mason or carpenter to copy, represents the exact lengths and positions of the lines which form the outline of the object, whereas an artist, who wishes to make a picture of the machine or building, represents these outlines as they appear to the eye.

In Solid Geometry the *rays of light*, by means of which an object is seen, are supposed to be *parallel* and not to converge towards a point as they actually do and are made to do in a Perspective drawing.

In order to represent a solid object upon paper by means of Solid Geometry, the draughtsman has to make two distinct drawings, viz.:

1st.—**The Plan** *of the object*, which is a drawing to represent the *exact space it covers*, or in other words the exact space it *overhangs*, as an eye looking directly on it from above would see it.

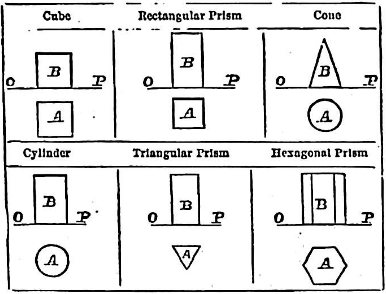
2nd.—**The Elevation** *of the object*, which is a drawing to represent its *vertical appearance*, that is, the exact space covered by the perpendicular height of the object, as an eye directly in front would see it.

Solid Geometry, therefore, enables us to represent the three dimensions of a solid, namely, *length*, *breadth*, and *height*, in two drawings. In one of these—*the Plan*—the length and breadth are shown, and in the other—*the Elevation*—the length and height.

The following illustrations will readily explain to the pupil what is meant by Plan and Elevation :—

A is the Plan of each of the objects, because it represents the exact space the object covers on a *horizontal plane*, say the floor of a room.

B is the elevation of each of the objects, because it represents the exact space the object covers on a *vertical plane*, say the wall of a room.



These drawings however convey no idea of a cube, prism, cone, &c., to the mind of the pupil who has never been instructed in the principles of Solid Geometry; the *plan of the cube* conveys no other idea than that of a square surface, and the *elevation* does the same, whilst the *plan of the cone* is simply a circular surface, and the *elevation*, a triangular one. The lines used in drawing the plans and elevations of solid objects are merely the lines which project the various points of the solid, whereas the form and position of the objects themselves, which stand in space, are to be conceived by the pupil. The power by means of which he is able to possess this conception can only be acquired by a knowledge of the principles on which solid objects are projected. These we are about to consider.

Two Planes.—The pupil must not forget that although A—*the plan*—and B—*the elevation*—are here drawn upon one plane—the same sheet of paper—still they represent the objects as covering two plane which are at right angles to each other. The *plan* covers the floor,

which we call the "**Horizontal Plane**" (H.P.), and the *elevation* covers the wall, which we call the "**Vertical Plane**" (V.P.) Thus whenever we speak of "*the plan*" of an object, the pupil must at once remember that we mean that view he would have by looking down on it; and when we speak of "*the elevation*," we mean that view he would have by looking at it directly in front.

Line of Intersection.—In constructing the plan and elevation of an object on one plane—the same sheet of paper—it is convenient to separate that portion of the paper which contains the plan from that which contains the elevation. This is done by means of a line such as the line O P in the foregoing illustrations, which is called the "**Line of intersection,**" and sometimes the *ground line*, or *base line*. If we take the *floor of a room* to illustrate the *horizontal plane* and the *wall* the *vertical plane*, the *line of intersection* is illustrated by the line in which the floor and wall intersect each other. The intersecting line throughout this treatise will be named O P. The contraction H.P. will stand for *horizontal plane*, and V.P., the *vertical plane*.

Projections.—The drawings which represent the plan and elevation of an object are called the "**Projections**" of that object, and hence we shall speak of "*Projecting* an object or projecting the plan and the elevation of an object. Having explained the use of Solid Geometry and the meaning of plan and elevation, we will now proceed to consider the principles upon which plans and elevations are projected.

The models most commonly used to illustrate these are the *cube*, *pyramid*, *prism*, *cylinder*, *cone*, and *sphere*. The teacher should also get two plane surfaces of wood, so constructed as to work on hinges, to illustrate the horizontal and vertical planes. We will suppose he has such a model, and in order to give the pupil every assistance, the drawings used in the first few problems will show the objects themselves which we wish to project, as placed on such a model, and also the manner in which the projections are obtained, thus—

Fig. A. represents a drawing of the object itself, and shows the methods of projection, whilst

Fig. B. represents the projections as they are actually drawn in practice.

In the first seven problems **1 2 3 4** *of Fig. A.* represents the V.P. as the wall of a room; **2 3 6 5**, the H.P. as the floor of the room; and O P the **Line of intersection.**

SECTION I.

PARALLEL PROJECTION.

PROJECTION OF POINTS AND LINES.

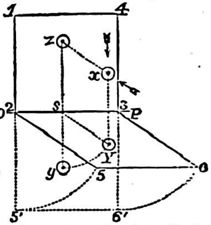
PROBLEM I.

To draw the plan and elevation of a **point suspended in space.**

Fig. A explains the *Theory*; Fig. B shows the *Practice*.

Note.—*The pupil may use a* pea *or* marble *as a tangible illustration of the point here indicated by x.*

Fig. A.

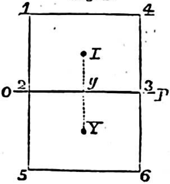


The Plan.—If we suppose the eye to see the point *x* directly from above, then the line *x* Y falling on the H.P. in the point Y will indicate the line of sight, and Y is the plan of the point *x*, since it is the point on the H.P. which *x* overhangs.

The Elevation.—If we now suppose the eye to see the point *x* directly in front, the line *x* Z falling on the V.P. in the point Z, will indicate the line of sight in that direction, and Z is the elevation of the point *x*, since it is the point on the V.P. which *x* covers when seen directly in front.

Note.—The lines *x* Y and *x* Z are called **Projectors**, because they indicate the line of sight, and are therefore used to project the plan and elevation. The projector *x* Z which is drawn from *x* at right angles to the V.P. represents the distance of the point *x* from the V.P., and the projector *x* Y, drawn at right angles to the H.P., the height of the point *x* above that plane.

Fig. B.



Let us now revolve the H.P. to 2 3 6′ 5′ to coincide with the V.P. 1 2 3 4, that is, until the two planes fall into the same plane. Then the Plan Y

will also revolve with it and fall into the position indicated by the point *y*, which is the position of the plan of such a point as shown in actual practice.

In Fig. B, I is the elevation and Y the plan of a point suspended in space, projected according to the principles explained in describing Fig. A.

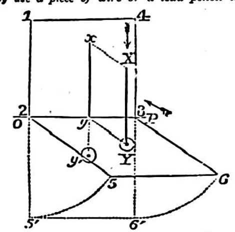
PROBLEM II.

To draw the plan and elevation of a line at right angles to the horizontal plane, and parallel to the vertical plane.

Fig. A explains the *Theory*; Fig. B shows the *Practice*.

Note.—*The Pupil may use a piece of wire or a lead pencil to illustrate the line here indicated by X Y.*

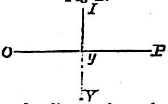
Fig. A.



The Plan.—The eye looking directly down on X Y, would see nothing but the point X, which covers the H.P. in the point Y, and is therefore the plan of the line.

The Elevation.—The line X Y as seen directly in front would cover the space marked by *x y* on the V.P., therefore *x y* is the elevation of the line, and its ends are determined by the projectors X *x* and Y *y*, which are drawn from X and Y at right angles. This however is not the method adopted by draughtsmen to illustrate the plan and elevation, it merely illustrates the manner in which we obtain these projections. See Fig. B.

Fig B.



Let us now revolve the H.P. 2 5 6 3 on the intersecting line O P, until it falls in a straight line, that is in the same plane with the V.P. 1 2 3 4. Then the point Y which revolves with the H.P. drops into the position marked *y'*, and is the plan of such a line as shown by

draughtsmen in actual practice. In Fig. B the point Y is the plan of a line at right angles to the H.P., and the line I y its elevation; the method of projecting these being shown in Fig. A.

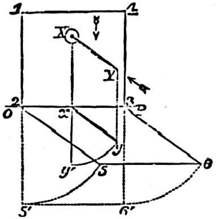
PROBLEM III.

To draw the plan and elevation of a line at right angles *to the* V.P. *and parallel to the* H.P.

Fig. A explains the *Theory*; Fig. B shows the *Practice*.

Let X Y represent the line, say a piece of wire, at right angles to the V.P., and parallel to the H.P.

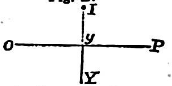
Fig. A.



The Plan.—An eye which views the line from above would see that it covers the space on the H.P. indicated by the line x y, which is therefore the plan of X Y.

The Elevation.—An eye which views the line directly in front would simply see the point Y which covers the V.P. in the point X. The point X therefore is the elevation of the line.

Fig. B.



Let us now revolve the H.P. as before until it falls in a line with the V.P and the line x y which revolves with the plane will fall into the position indicated by x y'. In Fig. B the point I is the elevation of a line at right angles to the V.P., and the line Y y is its plan, as drawn in actual practice; the method of projecting these being shown in Fig. A.

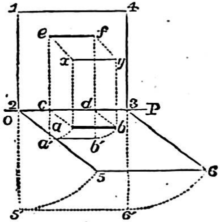
PROBLEM IV.

To draw the plan and elevation of a **line parallel to both planes** *and suspended in space.*

Fig. A explains the *Theory*; Fig. B shows the *Practice.*

Let x y represent such a line, say a piece of wire suspended in space.

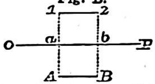
Fig. A.



The Plan.—Draw the projectors *x a* and *y b* of equal length, falling on the H.P. in *a* and *b*. Join *a b* which is the plan of the given line *x y*.

The Elevation.—The distance of *x y* from the V.P. is indicated by the distance of *a* and *b* from the intersecting line O P. Therefore draw the projectors *a c* and *b d* to the line O P; and also the projectors* *x e* and *y f* to the vertical plane to meet the perpendiculars drawn from *c* and *d* in the points *e* and *f*. These points determine the elevation of the points *x* and *y* on the V.P. Join *e f*, which represents the elevation of the given line *x y*.

Fig. B.



Let us now revolve the H.P. until it coincides with the V.P., and *a b* will assume the position of the line *a' b'* as it is shown in actual practice. In Fig. B, 1 2 shows the elevation, and A B the plan of a line parallel to both planes.

*Note.—The pupil must not lose sight of the fact that "*projectors*" are always drawn from the points projected at "*right angles*" to the planes of projection. In the Perspective Illustration, Fig. A, he will notice that the projectors *a c* and *b d* represent lines at right angles to O P, although they really fall at acute angles to that line, whereas in Fig. B we have a Geometric drawing of the plan and elevation in which the projectors A *a* and B *b* which correspond with the projectors *a c* and *b d*, Fig. 1, are really drawn at right angles to O P.

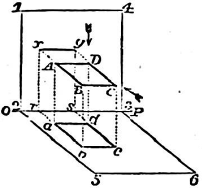
THE PROJECTION OF PLANES.

PROBLEM V.

To draw the plan and elevation of a **square surface** *suspended in space parallel to the* H.P. *and at right angles to the* V.P.

Fig. A explains the *Theory;* Fig. B shows the *Practice.*

Note.—*A drawing board may be used as an illustration* of the *square surface which is here indicated by A B C D,* parallel to the H.P., the edges A B and C D being at right angles to the V.P.



1st. *The plan must be a square of the same size.*

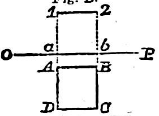
The Plan.—Draw the projectors B *b* and C *c* of equal length to determine the plan of the edge B C upon the H.P.* From *b* and *c* draw projectors *b r* and *c s* to O P, and let fall from A and D the projectors A *a* and *D d.* Join *a d.* Then *a b c d* 's the plan of the surface A B C D.

2nd. *The elevation must be a line equal in length to B C, the side of the square.*

The Elevation.—Now the elevation of B C on the plane 1 2 3 4 is determined by its distance from the V.P., and this distance is represented by the length of the projectors *b r* and *c s.* Therefore erect perpendiculars from *r* and *s* to intersect projectors drawn from A and D in *x* and *y.* Join the points of intersection *x* and *y.* Then the line *x y* is the elevation of the square surface A B C D.

Let us now revolve the H.P. 2 5 6 3 until it coincides with the V.P. 1 2 3 4, in doing which the plan *a b c d* must also revolve, and we shall get the projections of the square surface as they are shown in actual practice in Fig. B.

Fig. B.



This rotation is not shown in Fig. A to save confusion of lines

* The distance of *b c* from O P gives the distance of the edge B C from the V.P.

PROBLEM VI.

To project a square plane, *the surface of which is situated at right angles to the* H.P. *and parallel to the* V.P.

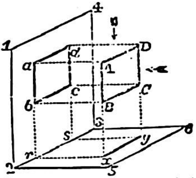
Fig. A explains the *Theory*; Fig. B shows the *Practice*.

1st. The plan must be a line.

The Plan—Draw the projectors B x and C y of equal length falling on the H.P. in x and y. Join x y and the line is the plan of the given plane surface A B C D.

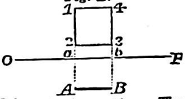
Draw the projectors x r and y s to determine the distance of A B C D from the V.P.

2nd. The elevation must be a square.



The Elevation.—Draw the projectors A a, B b, C c, and D d, to the vertical plane, the positions of a, b, c, d, being determined by projectors from r and s. Join a b, b c, c d, and d a, then a b c d is the elevation of the given plane surface.

Fig. B.



If we now revolve the H.P. 2 5 6 3 until it falls into the same plane as the V.P., we shall have the plan and elevation of such a square surface as shown in Fig. B, the way in which it is represented in actual practice. The rotation has not been made in Fig. A to save confusion of lines.

PROBLEM VII.

To project a cube *suspended in space with faces parallel to both planes.*

Let A F *represent a cube as it would appear suspended in space, in which the face* A B C D *is parallel to the* H.P., *and the face* C F G D *to the* V.P.

The operations involved here are similar to those already performed, because a solid is only composed of surfaces.

1st The plan must be a square.

The Plan—Project the plan a b c d of the surface A B C D, which is the only surface an eye would see when placed directly over the cube. (Prob. 5.)

2nd. The elevation must also be a square.

The Elevation—Project the elevation c' f g d' of the surface C F G D, which is the only surface an eye would see directly in front of the cube. (See Prob. 6.)

Fig. A.

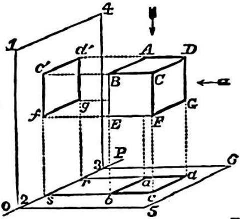
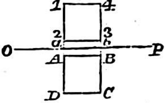


Fig. B.



If we now revolve the H.P. until it coincides with the V.P., we shall get the plan and elevation of the given cube as it would be actually drawn in practice, and is here represented in Fig. B, in which 1 2 3 4 is the elevation, and A B C D the plan.

The Pupil is now familiar with all the elementary principles affecting the projection of *points* and of *lines* and *surfaces* which are vertical to one of the planes of projection and parallel to the other, and also of solids, which are in the same position with respect to the two planes. Hence we shall dispense with the use of such illustrations as we have given in Fig. A of the foregoing Problems to illustrate the principles of projection, and shall apply these principles to the projection of various kinds of solid objects and in various positions. Before doing so, however, it may be well for the pupil to revise what we have already explained, and to note that in Solid Geometry—

1.—Two projections *are made of an object, one on the* H.P. *and another on the* V.P.

2.—The lines used to project the points of an object are called Projectors.

3.—Projectors are always drawn at *right angles* to the planes of projection.

4.—The projections of a point are represented by a point on both planes.

5.—The projection of a line at right angles to the H.P. and parallel to the V.P., is a *point* on the H.P. and a line of equal length on the V.P.

6.—The projection of a line at right angles to the V.P. and parallel to the H.P. is a point on the V.P., and a line of equal length on the H.P.

7.—The projection of a line parallel to both planes is a *line of equal length* on both planes.

8.—All *solids* are composed of *surfaces*, all *surfaces* of *lines*, and all *lines* are determined by *points*, therefore the projection of solids simply resolves itself into the projection of the points, lines, and surfaces of which the solids are composed, and is therefore governed by the principles we have just been considering.

QUESTIONS.

1. Draw the plan and elevation of a point suspended in space, 1½ inch from the V.P. and 2 inches from the H.P.

2. Project a line 2¼ inches long when parallel to the H.P. and at right angles to the V.P.; its height above the ground being 2 inches, and distance from the V.P. 1¾ inch.

3. Show the elevation* of a line 1½ inch long parallel to both planes, and 1½ inch from them.

4. Project a piece of straight wire 4 feet long, which is fixed in the wall at right angles to it, and 8 feet above the ground to which it is parallel. *Scale 2 feet to an inch.*

5. Draw the plan and elevation of a black board 4 feet square, suspended 2 feet above the floor of a school-room to which it is parallel and against the wall. *Scale 2 feet to an inch.*

6. Show the elevation* and plan of any cube having faces parallel to both planes.

* Note—the plan is to be projected from the elevation.

MISCELLANEOUS PROJECTIONS.

THE PROJECTION OF PRISMS.

A **Prism** is a solid object, the ends of which are *equal* and *similar surfaces*, and the sides which unite these ends are parallelograms.

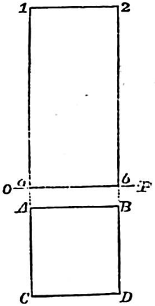
The ends of prisms may be either **triangles, squares,** or **polygons.**

PROBLEM VIII.

To project a **square prism,** *one end of which rests on the* H.P. *and one of its upright faces is parallel to the* V.P. *The height of the long edges is* 10′, *and of the end edges* 5′. *Scale* $\frac{1}{8}''$ *to the foot.*

10′ on this scale will be represented by $1\frac{2}{8}''$.
5′ " " " " " $\frac{5}{8}''$.

1*st. The plan of this prism will be a square.*



The Plan.—Draw A B $\frac{5}{8}''$ long, parallel to O P. On A B construct a square A B C D which is the plan of the prism.

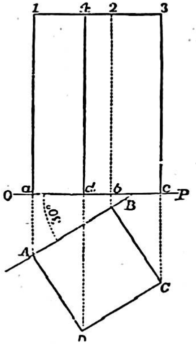
2*nd. The elevation of this prism will be a rectangle.*

The Elevation.—Draw projectors A *a* and B *b*, and at *a* and *b* erect perpendiculars *a* 1 and *b* 2, $1\frac{2}{8}''$ long, to represent the height 10′. Join 1 2 by a line parallel to O P; then the rectangle 1 *a b* 2 is the elevation of the prism.

Note.—The pupil may here observe that the distance of the parallel A B from the intersecting line O P really represents the distance of the prism from the V.P.; and also if the base of the prism were raised above the H.P., its distance would be represented by the length of a perpendicular let fall from that base to the line O P; for example the perpendicular 1 *a* represents the height of the line 1 2 above the H.P.

PROBLEM IX.

To project a prism *of the same dimensions with one of its faces inclined at an angle of* 30° *to the* V.P.



The pupil will perceive that although one of the faces of the prism is inclined at an acute angle to the V P., still the *axis** of the prism is parallel to it.

1*st. The plan, as before, is a square.*

The Plan.—Make A B 1½" long, at an angle of 30° to O P. On A B construct a square which is a plan of the prism.

2*nd. The elevation of the prism will be two seen and two unseen rectangles.*

The Elevation.—Draw the projectors A *a*, B *b*, C *c*, and D *d*., and at *a b c d* erect perpendiculars *a* 1, *b* 2, *c* 3, *d* 4, each 1¾" high. Join these ends by a line parallel to O P. and the elevation is complete.

An Analysis of the elevation.—The rectangle 1 *a* *d* 4, is the elevation of the face which rises up from the edge A D; 4 *d* *c* 3, of the face which rises up from C D; 1 *a* *b* 2 of the hidden face which rises up from A B, and 2 *b* *c* 3 of the hidden face which rises up from B C

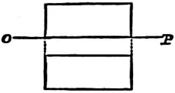
Note.—The same object may have a variety of plans and elevations, for these depend upon the position in which the object is viewed. For instance, the square prism we have just been projecting may have rested on the H.P. with one of its long faces parallel to the V.P. in which case the *plan* would have been a *rectangle* and the elevation a *rectangle* as well, or it may have rested on the H.P. with an end parallel to the V.P. and then the plan would have been a rectangle and the elevation a square, which is just the reverse of the projections given in Problem 8.

Problems for the pupil to solve are given in various ways. The *plan* of the figure may be given from which the *elevation* has to be projected, or the elevation may be given from which the plan has to be projected; but the pupil must also bear in mind that the solid, of which the given projection is a plan or elevation, may not be described in words, but by means of another view of the same solid.

* A line drawn from the centre of one end of a prism to the centre of the other end is called its axis.

OTHER PROJECTIONS OF THE SQUARE PRISM.

FIG. A.



Reverse the position of the prism on the model, and the pupil will observe that Fig. A shows the projection of a square prism, resting on the H.P. with one of its long faces parallel to the V.P.

FIG. B.

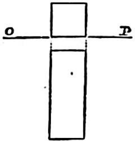


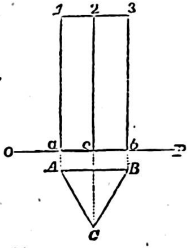
Fig. B shows an end elevation of the same prism when it rests on the H.P. with one of its long faces at right angles to the V.P.

Note.—The pupil may further extend these principles by projecting the square prism resting on the floor with one of its long faces at various angles to the V.P., always supposing that the axis of the prism is parallel to the H.P. Lines, however, inclined to the planes of projection will be considered in Section III.

PROBLEM X.

To project a **triangular prism** *resting on one of its ends, and having one of its faces parallel to the* V.P; *its height being* 8', *and the width of each of its triangular edges* 4'. (Scale 1/8 of an inch to a foot).

8' on this scale will be represented by 1"
4' " " " " " ½"



1st. *The plan is an equilateral triangle.*

The Plan.—Draw A B ½" long parallel to the intersecting line, and on it construct an equilateral triangle A B C. Then the triangle A B C is the plan.

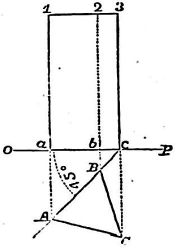
2nd. *The elevation is two rectangles.*

The Elevation.—Draw the projectors A *a*, B *b*, and C *c*, and erect the perpendiculars *a* 1, *c* 2, and *b* 3, equal to the height of the prism, viz.:—an inch. Draw a line through the points 1 2 3, parallel to O P, and the elevation is complete.

PROBLEM XI.

To project the **triangular prism** *given in* Prob. X, *having one of its faces inclined to the* V.P. *at an angle of* 45°.

Note.—*Although the faces are inclined to the V.P. its axis is parallel to that plane.*



1st. The plan is an equilateral triangle.

The Plan.—Draw A B ½″ long at an angle of 45° to O P. On A B construct an equilateral triangle A B C, which is the plan of the prism.

2nd. The elevation represents one seen and two hidden rectangles.

The Elevation—Draw the projectors A *a*, C *c*, and B *b*, and erect the perpendiculars *a* 1, *c* 3 and *b* 2, each 1″ high. Through 1, 2, 3, draw a parallel to O P and the elevation is complete. The edge *b* 2 is dotted, because it is hidden.

Note.—The rectangle 1 *a c* 3, is the elevation of the front face, of which the line A C is the plan; 1 *a b* 2 of the hidden face of which A B is the plan; and 2 *b c* 3 of the hidden face of which B C is the plan.

OTHER PROJECTIONS OF A TRIANGULAR PRISM.

Fig. A. Fig. B.

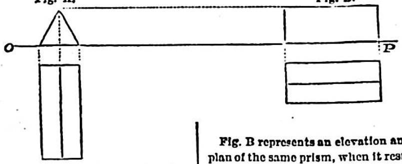


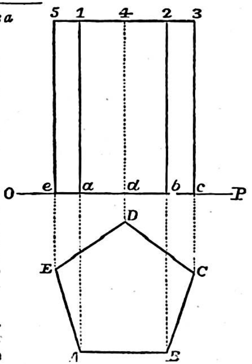
Fig. A represents the end elevation and plan of a triangular prism when it rests on one of its long faces at right angles to the V.P.

Fig. B represents an elevation and plan of the same prism, when it rests on the same face, with its long edges parallel to the V.P. The vertical height of the elevation is taken from Fig. A.

PROBLEM XII.

To project a **pentagonal prism** *resting on the* H.P. *on one of its ends; the front face being parallel to the* V.P. *Take the height of the prism as* 10′, *and width of each face* 5′. *Scale* ⅛ *inch to a foot.*

10′ on this scale is represented by 1¼″
5′ „ „ „ „ ⅝″



1st. The plan of this prism is a regular pentagon.

The Plan.—Draw A B ⅝″ long parallel to O P, and on it construct a regular pentagon A B C D E. This is the plan.

2nd. The elevation will represent 3 visible and 2 hidden rectangles.

The Elevation.—Draw the projectors A *a*, B *b*, C *c*, D *d*, and E *e*. From *a*, *b*, *c*, *d*, and *e*, erect perpendiculars *a* 1, *b* 2, *c* 3, *d* 4, and *e* 5, each 1¼″ high. Through these points draw a line which will be parallel to O P, and the elevation of the pentagonal prism is complete.

An Analysis of the elevation—The rectangle 1 *a b* 2, marks the elevation of the face of which the line A B is the plan. 5 *e a* 1 of the face which rises up from the edge E A; 2 *b c* 3 of the face which rises up from the edge B *c*; 5 *e d* 4 of the hidden face which rises up from the hidden edge E D, and 4 *d c* 3 of the hidden face which rises up from the hidden edge D C.

PROBLEM XIII.

A hexagonal prism *stands on the floor of a room having one of its faces inclined at an angle of* 45° *to the* V.P.; *the height of the prism being* 6′ *and the width of each face* 3′. *Scale* 6 *ft. to an inch.*

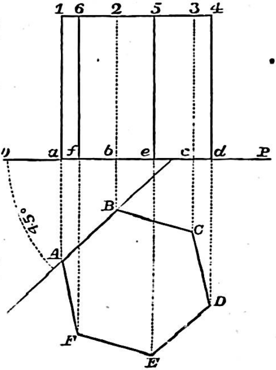
6′ on this scale will be represented by 1″
3′ „ „ „ „ „ „ „ ½″

1st. The plan is a regular hexagon.

The Plan.—Draw A B ½″ long and at an angle of 45° to O.P. On A B construct a regular hexagon A B C D E F which is the plan of the base and also of the upper surface.

2nd. The elevation will be 3 seen and 3 unseen rectangles

The Elevation.—From each of the angular points of the hexagon erect projectors A *a*, B *b*, C *c*, D *d*, E *e*, and F f. From each of the points a, b, c, d, e, f erect perpendiculars 1 *a*, 2 *b*, 3 *c*, 4 *d*, 5 *e*, 6 *f* each 1″ in length and through the points draw a line which will be parallel to O P, and complete the elevation.



Note.—The perpendiculars 2 *b*, and 3 represent the edges of the prism which are hidden, and are therefore indicated by a dotted line.

Analysis of Elevation.—The rectangle 1 *a f* 6 is the elevation of the seen face of which A F is the plan; 6 *f e* 5 of the seen face of which F E is the plan; 5 *e d* 4 of the seen face of which E D is the plan; 1 *a b* 2 of the hidden face of which A B is the plan; 2 *b c* 3 of the hidden face of which B C is the plan; and 3 *c d* 4 of the hidden face of which C D is the plan.

PROBLEM XIV.

Draw the plan and elevation of a **solid hexagonal column** *the height of which is* 30′ *and length of one side* 10′, *on a scale of* 20′ *to the inch; the front face being parallel to the* V.P.

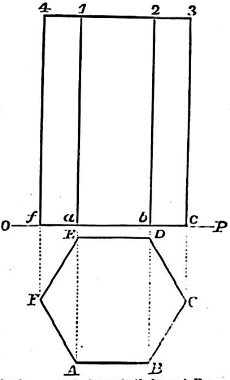
30′ on this scale will be represented by 1½″
10′ " " " " " " " ½″

1st. The plan will be a regular hexagon.

The Plan.—Draw A B ½" long parallel to O P, and on it construct a regular hexagon, A B C D E F, which is the plan.

2nd. The elevation is a series of rectangles.

The Elevation.—Draw the projectors A *a*, B *b*, C *c*, F *f*. From *a*, *b*, *c*, *f* erect perpendiculars 1 *a*, 2 *b*, 3 *c*, 4 *f*, each 1½" high. Through the numbered points draw a line which will be parallel to O P, and the elevation is complete.



Note.—The projectors from A and B, the angles at the base of the front face, are coincident with those which are drawn from E and D, and are therefore unseen.

The rectangle 1 *a b* 2 is an elevation of the front face which rises up from A B and also of the face which rises up from E D. The rectangle 4 *f a* 1 is the elevation of the faces which rise up both from A F and F E, and the rectangle 2 *b c* 3 of the faces which rise up from B C and D C.

PROBLEM XV.

A **Monument** *in the shape of a* **Pyramid** *stands on the floor of a church. The edges of the base are each 4′, and vertical height 10′. Two of its edges incline at an angle of 75° to the vertical plane, and the other two at an angle of 15°. Scale 8′ to 1 inch.*

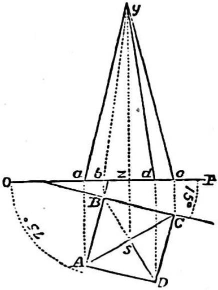
10′ on this scale will be represented by 1¼"

4′ " " " " " ½"

1st. The plan will be a square.

The Plan—Draw the line A B at an angle of 75° to the line of intersection, and ½ an inch long. On A B construct the square A B C D. Then A B C D is the plan of the pyramid, in which A B and its parallel D C are both inclined at an angle of 75° to O P; and A D and its parallel B C are each inclined at an angle of 15°.

2nd. The elevation will represent two seen and two unseen triangles.

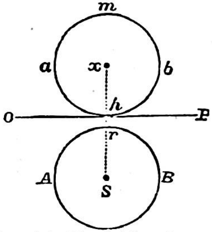


The Elevation—The vertical height or axis of a pyramid is measured by a perpendicular drawn from the centre of the base to its apex. Find S the centre of the base, and draw the projector S *z*. From *z* erect a perpendicular *z y* $1\frac{1}{4}$" high, which represents the height of the elevation. Draw the projectors A *a*, D *d*, C *c*, and B *b* from the angular points of the square. Join *a y*, *b y*, *c y*, and *d y* which represent the oblique edges of the pyramid. Then the triangle *y a c*, and the lines enclosed represent the elevation of the pyramid.

Note—The edge *b y* is hidden from the spectator, therefore it is marked in dotted lines. The face of the pyramid rising from **A D** is represented by the triangle *y a d*; that rising from D C by the triangle *y d c*; the hidden face rising from A B by the triangle *y a b*, and the other hidden face rising from B C by the triangle *y b c*.

PROBLEM XVI.

A Globe, 9" *in diameter, stands on a square table, the edge of which touches the wall of a schoolroom. Draw its plan and elevation on a scale of* 1' *to* 1".



Note—The top of the table may be taken as the H.P. and the wall the V.P.

9" on this scale will be represented by $\frac{3}{4}$ of an inch.

1*st. The plan is a circle.*

The Plan—Take any point S below O P as centre, and describe a circle, A B having for its diameter $\frac{3}{4}$ of an inch. Then the circle A B is the plan of the sphere.

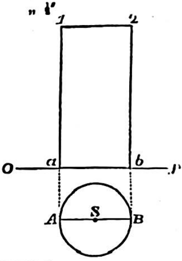
2*nd. The elevation is a circle.*

The Elevation—The centre of the circle required for the elevation is obtained by a projector drawn from S. Draw the projector S *h* and at *h*, erect a perpendicular *h x*, equal in length to the radius *s r*. From centre *x*, describe the circle *a b*, which is the elevation of the sphere.

PROBLEM XVII.

To project a cylinder *resting on the* H.P. *on one of its ends, its height is 4′, and the diameter of its base 2′. Scale ¼′ to the foot.*

4′ on this scale will be represented by 1″
2′ " " " " " ½″



1st. *The plan of this cylinder is a circle.*

The Plan—Take any point S below O P as centre, and describe a circle, having ½″ for its diameter. This is the plan.

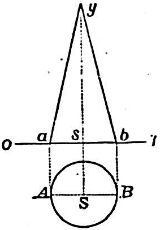
2nd. *The elevation is a rectangle.*

The Elevation—Draw the diameter A B of the plan parallel to O P, and throw up the projectors A *a* and B *b*. From *a* and *b*, erect the perpendiculars *a* 1 and *b* 2, each 1″ high. Join 1, 2 by a line which is parallel to O P. Then the rectangle 1 *a b* 2 is the elevation of the cylinder

PROBLEM XVIII.

To project a cone *resting on the* H.P., *its vertical height being 4′, and the diameter of its base 2′. Scale ¼″ to the foot.*

4′ on this scale will be represented by 1″
2′ " " " " " ½″



1st. *The plan of the cone is a circle.*

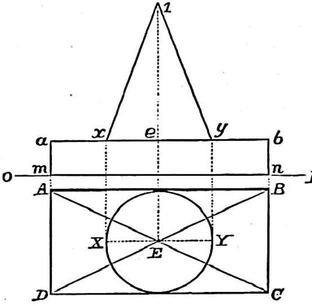
The Plan—Take any point S below O P as centre, and describe a circle having ½ inch for its diameter. This is the plan.

2nd. *The elevation is an isosceles triangle.*

The Elevation—The apex of the cone stands directly over the centre of its base, therefore draw the projector S *s* and at *s* erect the perpendicular *s y*, 1″ high. Then *s y* is the axis of the required cone. Draw the projectors A *a* and B *b*, and join *y a* and *y b*, then the triangle *y a b* is the elevation of the cone.

PROBLEM XIX.

A **rectangular block** *of stone 6′ long, 3′ wide, and 1′ high, rests on a level floor on one of its longest and widest faces, parallel to the* V.P. *On the upper surface a* **cone** *rests on its base, the axis of which is 4′. The diameter of the base of the cone is 3′ and the centres of the block and cone are coincident. Draw a plan and elevation of both block and cone. Scale ¼″ to a foot.*



The Plans.—*1st. The Block.* Draw a line A B 1½″ long parallel to O P, and construct a rectangle A B C D making the length of the other parallel sides A D and B C, each ¾″. Then A B C D is the plan of the rectangular block.

2nd. The Cone. Find E, the centre of the plan of the block, which is also the centre of the cone that rests on the block. Since the diameter of the cone and the width of the block are each 3′, from E describe a circle X Y touching the long edges A B and D C of the block. Then the circle X Y so described is the plan of the cone.

The Elevations.—*1st. The Block.* Draw the projectors A *m* and B *n* of the block, and erect perpendiculars *m a* and *n b* equal in length to the height of the block, viz., 1′ that is on this scale ¼″. Join *a* and *b* by a line which is parallel to O.P. Then *a b n m* is the elevation of the block.

2nd. The Cone. From E draw the projector E *e*, and at *e* erect the perpendicular *e* 1 an inch high, which will be the axis of the cone. Draw a diameter of the plan X Y parallel to O P, and from X and Y erect projectors X *x* and Y *y*; join 1 *x* and 1 *y*, and then the triangle 1 *x y* is the elevation of the cone.

PROBLEM XX.

To Project a Tetrahedron *of any dimensions, with one of its faces resting on the* H.P. *and one side inclined at an angle of* 45°

Note—*A Tetrahedron is a solid with four faces, and each face is an equilateral triangle.*

Fig. A.

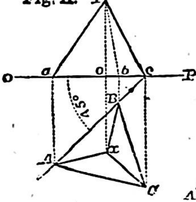
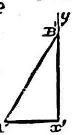


Fig. B.



The Plan—Draw A B at an angle of 45° to the line of intersection. On A B construct an equilateral triangle A B C, which is the plan.

The Elevation—The vertical height or axis of a tetrahedron is a line drawn at right angles to the base from its centre. Therefore find x the centre of the triangle, which is the plan of the axis. Join A x, B x, and C x, and the lines so drawn are plans of the edges of the solid. The height of the axis depends upon the length of its edge for it forms one side of a right angled triangle, the other sides of which are an edge and a plan of that edge. Hence it is necessary to construct an especial right angled triangle to obtain the axis, this is done in Fig. B. Make a line A' x' equal to A x. At x' erect a perpendicular x' y of any length; and from A' as centre, ith radius A B cut off A' B'. Then x' B' is the height of the axis of the tetrahedron. Draw the projector x o and from o draw the perpendicular o 1, equal in length to x' B'. From A, B, and C draw the projectors A a, B b, and C c. Join a 1, b 1, c 1, and the triangle 1 a c is the elevation of the tetrahedron.

N.B.—The elevation of the edge 1 b cannot be seen, it is therefore marked in dotted lines. 1 a b is the elevation of the hidden face of which A B is the plan; and 1 b c of the hidden face of which B C is the plan.

QUESTIONS.

1. Project a pentagonal prism which stands on the floor on one of its ends, having a hidden face parallel to the H.P. Height 3 inches, width of face 1½ inch.
2. Draw the elevation and plan of a cone, the height of the elevation being 2 inches and the width of the base 1½ inch.

Note—*Draw the elevation first and the plan afterwards.*

3. The elevation of a square pyramid is simply an isosceles triangle, the height of which is 3 inches, and the width of its base 1½ inch. Draw it and project its plan ½ an inch from the V.P.

SECTION II.

ANGULAR PROJECTION.

INCLINATION TO ONE PLANE.

We have hitherto explained how solids are projected which have had their bases parallel to one of the planes of projection, whilst either their faces or their axes have been parallel to the other plane. We will now consider how a solid can be projected, whose base or axis may be inclined at an angle to one of the planes of projection.

But since solids are bounded by surfaces and surfaces by lines, let us commence with the projection of inclined lines, and the two illustrations which follow will give the pupil a ready conception of the manner in which lines and surfaces may be placed at an angle to one of the planes.

Fig. 1.

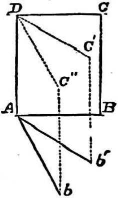


Fig. 2.

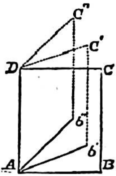


Fig 1 *gives the pupil a notion of surfaces inclined at an angle to the* V.P.

If A B C D is the elevation of a surface parallel to the V.P. and at right angles to the H.P., then

A *b′ c′* D may represent the same surface inclined at a small angle to the V.P., and A *b c″* D the same surface inclined at a greater angle to the V.P., but both surfaces are at right angles to the H.P.

Fig. 2 *gives the pupil a notion of surfaces inclined at an angle to the* H.P.

If A B C D is the plan of a surface parallel to the H.P. and at right angles to the V.P., then

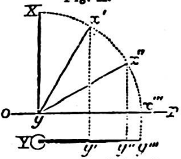
A b' c' D may represent a surface inclined at a small angle to the H.P., and A b'' c'' D the same surface inclined at a greater angle to the H.P. but both surfaces are at right angles to the V.P.

With this general notion of lines and surfaces inclined to the planes of projection, let us now proceed to the projection of inclined figures.

PROBLEM XXI.

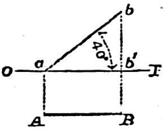
To project a line inclined at an angle to the H.P. (*say of* 60° *or* 30°) *but parallel to the* V.P.

Fig. A.



From Prob. 2 we learn that the line X y is the elevation of a piece of wire at right angles to the H.P., and point Y is the plan of such a line. Let us now suppose the piece of wire to revolve on the pivot Y, that is on y the point of its projection on the line of intersection, so as to be inclined at an angle of 60° to the H.P. as represented on the elevation by $y\,x'$, and also at an angle of 30°, as represented by $y\,x''$, but still keeping its parallel position to the V P. Then an eye looking from above on the line $y\,x'$, which is inclined at an angle of 60° to the H.P., would see that its plan is the line Yy', parallel to the line of intersection, and an eye looking down on $y\,x''$ which is at an angle of 30° to the H.P., would see that its plan is the longer line Yy'', and if the wire be still rotated until it falls parallel with the H.P. as indicated by $y\,x'''$, its plan is the line $y\,y'''$ the full length of the wire. From this illustration we may make two deductions.

Fig. B.



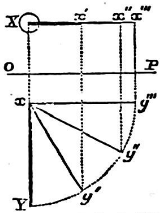
1st. *The* plan of a line *inclined at an angle to the* H.P. *is a line parallel to* O P., *and its length varies with the size of the angle at which it is inclined—the greater the angle of inclination the shorter will be the plan.*

2nd. *The* elevation of a line *inclined at an angle to the* H.P. *and parallel to the* V.P. *is a line at the given angle and of the given length.*

Fig. B represents the *elevation* and *plan* of a line inclined at an angle of 40° to the H.P.

PROBLEM XXII.

To project a line **inclined at an angle to the V.P.** *(say of* 60° *or* 30°*), but parallel to the* H.P.



From Problem III we learn that a line falling into the position of x Y is the plan of a line (illustrated by a piece of wire) at right angles to the V.P., and parallel to the H.P. and that X is the elevation of such a line. Let us now suppose that the wire revolves on x by means of a hinge to angles of 60° and 30° respectively; then an eye looking down on it in such positions would see that $x\ y'$ is the plan of a line at an angle of 60° to the V.P., and $x\ y''$ is the plan of a line at an angle of 30° to the V.P. Also an eye looking at this line in front would see that the straight line X x' is the elevation of a line inclined at 60° to the V.P., but parallel to the H.P., whilst X x'' is the elevation of the line when inclined at an angle of 30°; and if we rotate the line so as to be parallel to the V.P., then X x''', the full length of the wire, is its elevation. From this illustration we may also make two deductions.

1st. *That the* **plan of a line** *inclined at an angle to the* V.P. *but parallel with the horizontal plane, is a line at an angle of corresponding size and of the same length.*

2nd. *That the* **elevation of such a line** *is a straight line parallel to* O P, *which varies in length with the size of the angle, and that the greater the angle the less will be the length of the line of elevation.*

Fig. B.

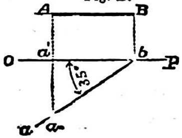
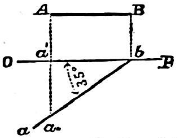


Fig. B represents the projections of a line inclined to the V.P., at an angle of 35°.

a'' b is its plan.
A B *is its elevation.*

PROBLEM XXIII.

A B is the elevation of a line *3 feet long, which is parallel to the* H.P., *but inclined to the* V.P. Project its plan *and determine the angle at which it is inclined.* (*Scale ¼″ to the foot.*)



The pupil has just learnt that the plan of a line parallel to the H.P. but inclined to the V.P. is a line equal to its entire length and inclines at the given angle. Therefore from A and B drop projectors A a' and B b to O.P.

Since the elevation A B represents 3 feet, it represents on this scale a line which on the plan will be ¾″ long. Therefore from b, with radius ¾″, mark off on A a' produced the point a''. Then a'' b is the plan of the line, and the angle a' b a'' is the angle of inclination to the V.P., which the pupil will find by means of his protractor is an angle of 35°.

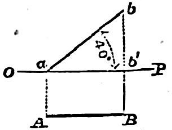
NOTE.—It may be as well to remind the pupil at this stage—

1st. That the elevation of an object, as A B in this figure, may be situated any distance above O.P. the line of intersection; and that the position it occupies simply represents the height of the object above the horizontal plane.

2nd. That the plan of an object, as the line a'' b in this figure may be situated any distance from the line of intersection O.P., and that the position it occupies simply represents the distance between the object and the vertical plane.

PROBLEM XXIV.

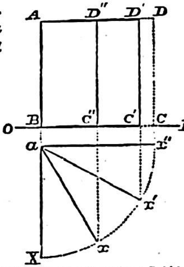
A B is the plan of a line 3 ft. long, which is parallel to the V.P. *but inclined to the* H.P. Project its elevation *and determine the size of the angle of inclination.*—(*Scale ¼″ to the foot.*)



Draw the projector A a and the point a may be taken as the end of the line A B on the line of intersection. Take a radius ¾ of an inch, which on the scale given represents 3 feet, and from a with this radius mark off the point b on the projector B b' produced. Join a b, then a b is the elevation of the line, and the angle b a b' is the angle of inclination to the H.P., which the pupil will find by his protractor is an angle of 40°.

PROBLEM XXV.

To project any **square surface,** *say a door, when* **opened at angles of 30°** *and* **60°** *to the wall,* **(that is to the V.P.)**



It is clear that A B C D is the elevation of a square plane, say a door, parallel to the V.P. and at right angles to the H.P. and also that $a\ x''$ is the plan of such a plane.

Let us now suppose the whole door to revolve to an angle of 30° to the V.P., that is, into the position indicated by the line $a\ x'$. Then it is clear that $a x'$ is the plan of the door in this position.

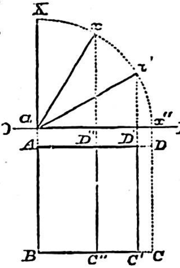
From x' draw a projector $x'\ C'$, and erect the perpendiculars C' D'. Then the elevation of the door, inclined at an angle of 30° to the V.P. is represented by A B C' D'

If we still rotate the door until it is inclined at an angle of 60° to the V.P. and yet maintains its parallel position to the H.P., it is quite evident that its plan will be the line $a\ x$, and its elevation the rectangle A B C'' D''.

Note.—And if we still rotate the door until it is at right angles to both planes of projection, then both plan and elevation will be straight lines, viz.:—a X the plan and A B the elevation.

PROBLEM XXVI.

To project a **square trap door** *in the floor, when opened on its hinges at* **angles of 30° and 60°,** *or any other number of degrees, to the floor, (that is to the* **H.P.)**

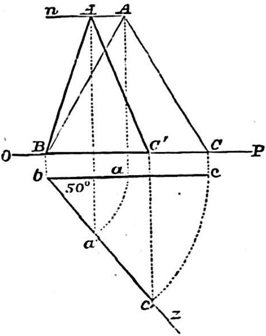


In Prob. V. we see that a square falling into the position of A B C D is the plan and a x'' the elevation of a plane at right angles to the V.P. and parallel to the H.P. Let us now suppose that this plane (a trap door) revolves on A B until it reaches angles of 30° and 60° to the H.P; the edges A D and B C still maintaining their parallel position to the V.P. Then it is quite clear that $a\ x'$ represents the elevation of the plane at an angle of 30° to the H.P., and $a\ x$ the elevation of the plane at an angle of 60° to the H.P.

And looking down on the door, it is also evident that A B C' D' is the plan of the door when opened at an angle of 30° to the H.P., and A B C'' D'' is the plan of the door when opened at an angle of 60°.

PROBLEM XXVII.

To project an **equilateral triangle,** *its surface being inclined at an* **angle of 50°** *to the* V.P., *with its base parallel to the* H.P.



The Plan—Draw the triangle A B C in thin lines to represent its elevation, parallel to the V.P.; *b c* being its plan in that position. Now the projection of the triangle at an angle of 50° is simply the projection of the lines which bound it, and we have before seen in Prob. 22, *that the plan of a line inclined at an angle to the V.P., but parallel to the H.P. is a line of equal length, and at an angle of corresponding size.*

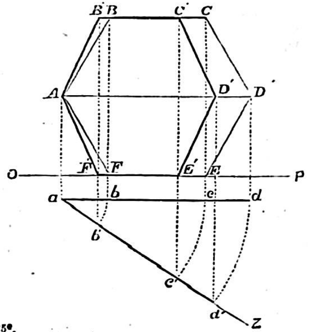
Hence from *b* draw a line *b z* of indefinite length at an angle of 50°, and from *b* as centre describe an arc *c c'*. Then *b c'* is the plan of the base B C, and therefore of the whole triangle, inclined at 50° to the V.P. Also describe the arc *a a'* from the centre *b* to get the position of *a* on the plan in its angular position. Then *a'* is the plan of the apex A in its angular position. The pupil must remember that although the triangle is inclined to the V.P. its base B C is still parallel to the H.P., and thus the height of its vertical projection is not altered. Therefore draw a line A *n* of indefinite length parallel to O P, and the elevation of A when at an angle of 50° is determined by the projector *a'* A'. The point *c'* which is the elevation of point C at an angle of 50° to the V.P. is also determined by the projector *c'* C'. Join points A' B and A' C', then the elevation of the whole triangle at an angle of 50° to the V.P. is represented by A' B C'.

Note.—It is not necessary that the elevation of the base of the triangle, that is B C should be on the line of intersection; in this position it merely represents the base as resting on the horizontal plane. It might be any distance above O P, so long as it keeps its parallel position to that line.

PROBLEM XXVIII.

To project **a regular hexagon** *at an* **angle of 35°** *to the* **V.P.** *its axis being parallel to the* **H.P.**

The Plan—Draw the hexagon ABCDEF in thin lines to represent the projection parallel to the V.P., and having its axis AD parallel to the H.P. Then *a d* is the plan of the axis; *b* is the plan of both B and F, and *c* is the plan of both C and E. From *a* draw *a z*, at an angle of 35° to *a d*, and from *a* as centre describe the arcs *d d′*, *c c′*, *b b′*. Then *a d′* is the plan of the axis A D at an angle of 35°; *b′* is the plan of B and F at angle of 35°, and *b′ c′* is the plan of both F E and B C at an angle of 35°.



The Elevation—A is still the elevation of the angle A, the angular position of the hexagon, but the position of the other points B and F, and C and E in the elevation at an angle of 35° are determined by projectors drawn from the points *b′* and *c′*, which are the plans of those points at that angle.

1st. Draw the projector *d′* D′. Then A D′ is the elevation of the axis A D at an angle of 35° to the V.P.

2nd. Draw the projector b′ F′, and produce it to meet B C produced in B′. Then B′ and F′ are the elevations of B and F at an angle of 35° to the V.P.

3rd. Draw the projector *c′* E′ and produce it to C′. Then E′ and C′ are the positions of E and C at an angle of 35° to the V.P.

Join A B′, A F′, E′ D′ and D′ C′, and the whole figure A B′ C′ D′ E′ F′ is the elevation of the given hexagon at an angle of 30°.

QUESTIONS FOR THE PUPIL TO ANSWER.

(1) What line in the above figure represents the plan of A B and A F, inclined at an angle of 35° to the V.P.

(2) What line in the above figure represents the plan of C D and D E, inclined at an angle of 35° to the V.P.

(3) What two lines in the above figure represent the elevations of the line *a b′*

PROBLEM XXIX.

To project a **rectangular prism** *when it rests on one of its solid angles, so that its axis* **inclines at an angle of 40°** *to the* **H.P.,** *but is parallel to the* **V.P.** *The width of its side is ½ inch, and its height 1½ inch.*

Note—It is convenient first of all to project the plan and elevation of the prism when its axis is parallel to the V.P. and at right angles to the H.P., as is done in Fig. 1. (See also Problem IX.)

Fig. 1. Fig. 2.

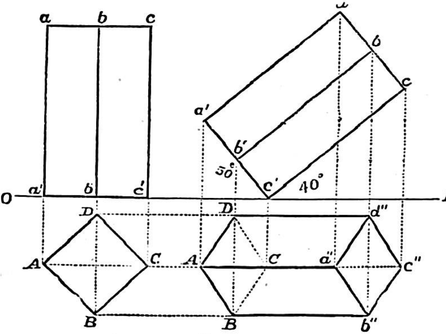


FIG. 2.

The Elevation.—It is quite evident that with the exception of the inclination of the prism its vertical projection is precisely the same as in Fig. 1, for it covers exactly the same vertical space. Therefore in Fig. 2, which shows the elevation of the solid inclined at an angle of 40° to the H.P., the pupil has simply to draw a line a' c', equal to c c' in Fig. 1, at an angle of 50° to O P,* which gives the inclination of the axis to the H.P., because in a square prism the edges are parallel to the axis. The remaining lines in the elevation of Fig. 1 have now to be copied, for the new elevation as shown in Fig. 2, and the elevation of the prism with its axis inclined at an angle of 40° to the H.P. is complete.

The Plan.—Now although the elevation retains its original shape as in Fig. 1, and merely differs in position, still the plan of the prism tilted on the solid angle C, becomes very much altered. For whilst in Fig. 1, the plan of the prism is simply a

* If the base a' c' make 50° with O P, then the axis b' b inclines at 40° to O P.

square; in Fig. 2 it is so placed that not only the top surface is seen, but also two of the adjacent faces which were hidden in Fig. 1; nevertheless, its axis is still parallel to the V.P., and therefore an eye looking down on the solid would see that the width of the diagonal of the base D B remains the same, and can therefore be determined in Fig. 2, by lines drawn parallel to O P from B and D, Fig. 1. The other diagonal of the plan, viz.: A C, has become shortened, because it is now inclined to the H.P. instead of being paralled to it. The plan of the upper surface *a b c* in Fig. 2 is found by drawing projectors from those points to meet the parallels drawn from A, B, C, and D, Fig. 1, in the points *a*″, *b*″, *c*″, *d*″ as shown in the illustration. The plan of the lower edges which the pupil must remember are still parallel to the upper ones, is also found by means of projectors drawn from a′, b′, c′ Fig. 2, to intersect the parallels drawn from A, B, C, and D, Fig. 1, in A, B, C and D, Fig. 2, as shown in the illustration. The lower edges D C and B C which are hidden are indicated by dotted lines. We have now found the plans of the upper and lower faces, and by joining the corresponding angles in these plans the pupil will find that the plans of the long edges are represented by the equal straight lines D *d*″, A *a*″, B *b*″, and C *c*″, and the plan of the whole prism inclined at an angle of 40° to the H.P. is completed.

ANALYSIS.

1st. In Fig. 2, the *lozenge a*″ *b*″ *c*″ *d*″ is the plan of the upper surface of the prism, which was represented in Fig. 1, by the *square* A B C D.

2nd. In Fig. 2, the *rhomboid* A *a*″ *b*″ B is the plan of the surface of which the *line* A B is the plan in Fig. 1.

3rd. In Fig. 2, the *rhomboid* A *a*″ *d*″ D, is the plan of the surface of which the *line* A D is the plan in Fig. 1.

PROBLEM XXX.

To project a hexagonal prism, *resting on one of its solid angles; its axis being inclined to the* H.P. *at an* angle of 65°, *but parallel to the* V.P. *Its height is* 10′ *and the width of each of its faces* 4′. *(Scale ⅛″ to the foot.)*

FIG. 1.

Fig. 1 shows the plan and elevation of the prism when its axis is parallel to the V.P., and at right angles to the H.P.

Note.—The base B C is ½″ long, which represents on the required scale 4′.
The axis *s x* is 1¼″ „ „ „ „ „ „ 10′.

FIG. 2.

Fig. 2 shows the plan and elevation of the prism when revolved on its solid angle A, until its lower face inclines at an angle of 25°, which makes its axis incline at 65°. The pupil will notice that in this position he sees the upper end and two of the faces of the prism when looking directly on it from above.

The Elevation.—The elevation is merely a copy of the elevation in Fig. 1.

The Plan.—The various points in the plan are determined by projectors dropped from the corresponding points in the elevation Fig. 2, to meet parallels to O P, drawn

Fig. 1.

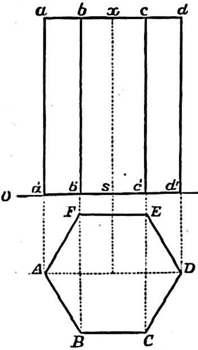
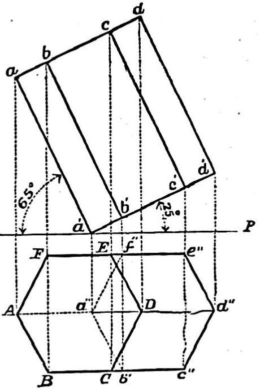


Fig. 2.



from the corresponding points in the plan of Fig. 1. The pupil should first draw the plan of the lower end of the hexagonal prism represented in the drawing by $a'' b'' c'' d'' e'' f''$. He should next draw the plan of the upper end of the hexagonal prism represented in the drawing by A B C D E F. Then the plans of the faces of the prism are drawn by simply joining the contiguous angles, which the following analysis explains.

Analysis.—C c'' is a plan of the edge $c\,c'$, and E e'' is a plan of the edge opposite C c', which was hidden in Fig. 1, its plan being represented in that Figure by the point E.

A a'' is a plan of the hidden edge $a\,a'$; B b'' is a plan of the edge $b\,b'$; and F f'' is a plan of the edge opposite b b', which is hidden in the plan of Fig. 1, its plan being represented in that Figure by the point F.

The hexagon A B C D E F is the plan of the upper end of the hexagon, of which A B C D E F is the plan in Fig. 1.

The rhomboid E D d'' e'' is a plan of the face which is hidden in Fig. 1, being represented in that figure by the line E D.

The rhomboid C D d'' c'' is a plan of a face which was also hidden in Fig. 1, being represented in that figure by the line C D.

The pupil may extend the analysis of the other lines and faces in the same manner

PROBLEM XXXI.

To project a **hexagonal pyramid,** *the diameter of the base being 6 ft. and the height* 10 *ft.*; *its axis being inclined at an angle of* 50° *to the* H.P. *but parallel to the* V.P. (*Scale* 8 *ft. to* 1".)

Note.—1st. The height 10 ft. on this scale will be represented by an axis line of $1\frac{1}{4}$ inch.

2nd. The diameter of the base 6 ft. will be represented by a line of $\frac{3}{4}$ inch.

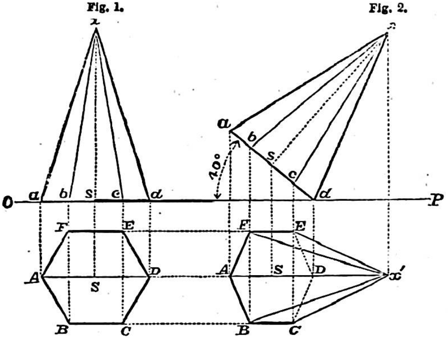


FIG. 1.

Fig. 1 shows the two projections of this pyramid when its axis is parallel to the V.P. and at right angles to the H.P. (See Problem XIV.)

FIG. 2.

The Elevation.—As in the case of the prism, the elevation of the pyramid inclined to the H.P. differs from the prism as shown in Fig. 1. in position only. Its shape and size are exactly the same as drawn in Fig. 1. Therefore place *a d* the elevation of the diameter in Fig. 1 at an angle of 40°, Fig. 2, which makes the inclination of the axis at 50°. Bisect *a d* in *s*, and erect the axis *s x* perpendicular to it. The elevation may now be completed by copying that in Fig. 1.

The Plan.—The projection of the various points for the plan of the prism are obtained by dropping perpendiculars from the various points in the elevation of Fig. 2, to intersect the parallels to O P, drawn from the corresponding points in the plan of Fig. 1. Hence the vertical projectors from *a*, *s*, *d* and *x* falling on the

parallel A D produced, determine the position of these points on the plan. The projector from b which meets F E and B C (Fig. 1.) produced, determines the position of the angles F and B on the plan, Fig. 2. And so the projector from C which meets F E and B C (Fig. 1.) produced, determines the positions of angles E and C in the plan, Fig. 2.

Join x', the plan of the apex of the pyramid, with A, B, C, D, E, F, the contiguous angular points in the base, and the plan of the whole pyramid is complete. The edges of the base E D and D C are in dotted lines, because they are concealed by the body of the pyramid.

The pupil will notice that the axis S x' and the diameter A D are in the same line, which is parallel to the V.P.

Analysis.—1st. Four faces are seen in the plan. x' E F is the plan of the face of which F E is the plan in Fig. 1. x' F A is the plan of the face of which F A is the plan in Fig. 1. x' A B is the plan of the face of which A B is the plan in Fig. 1. and x' B C is the plan of the face of which B C is the plan in Fig. 1.

2nd. Two faces are concealed in the inclined plan. x' E D is the plan of the hidden face of which E D is the plan in Fig. 1. x' C D is the plan of the hidden face of which C D is the plan in Fig. 1.

PROBLEM XXXII.

To project a cone *the axis of which is inclined at an angle of* 45 *to the* H.P., *but parallel to the* V.P. *The diameter of the base of the cone is* 8 *ft. and its vertical height* 10 *ft.* (*Scale* ⅛" *to the foot.*)

FIG. 1.

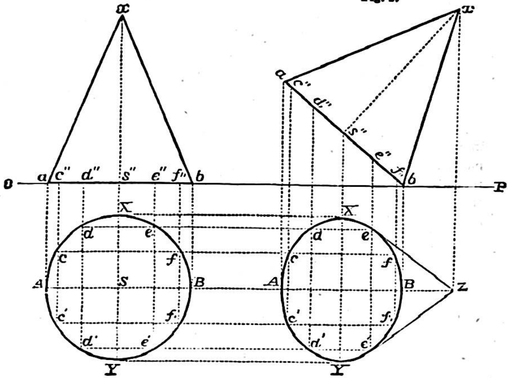
Fig. 1 shows the projections of the cone when its axis is parallel to the V.P., and at right angles to the H.P.; the plan being simply a *circle* and the elevation an isosceles *triangle*. (See Problem XVIII.)

NOTE.—$a\ b$ is 1" (8 eighths) long, which on the required scale represents 8'.
$s'\ x$ is 1¼" (10 eighths) high „ „ „ „ „ 10 ft.

FIG. 2.

The Elevation.—Draw $a\ b$ the elevation of the diameter of the cone at an angle of 45°, because when the axis is inclined at 45° the diameter of the base is also inclined at 45°. Transfer the elevation $a\ x\ b$ Fig. 1, to $a\ x\ b$ Fig. 2. Now draw a diameter A B on Fig. 1, parallel to O P, and another X Y at right angles to it. The length of the diameter A B on the plan Fig. 2 is determined by projectors dropped from a and b to meet the diameter A B Fig. 1 produced. The width of the diameter X Y on the plan Fig. 2, is the same as the width of the diameter X Y on the plan Fig. 1, and is determined by parallels to O P, drawn from X and Y Fig. 1, to meet a projector dropped from s', Fig. 2. The pupil will now observe that the plan of the circle, Fig. 1, at an angle of 45° as in Fig. 2, is an ellipse, of which X Y and A B are the transverse and conjugate diameters. He will at once ask himself how the points through which the curve of the ellipse passes are to be projected. The plan of the

Fig. 1. Fig. 2.



circle Fig. 1, has neither point nor angle for projection, and therefore the disc has to be prepared for such. On the plan Fig. 1, mark off any points *c c'*, *d d'*, *e e'*, *f f'*, on either side of A B, and equidistant from it. Join *c' c*, *d' d*, *e' e*, *f' f*, and produce the lines as projectors to *a b*, marking on it the points *c" d" e" f"*. Transfer these points to *a b*, the elevation of the diameter on Fig. 2, and from these transferred points let fall projectors to intersect the parallels to O P drawn from the corresponding points in the plan of Fig. 1. The curve of the ellipse passes through the points of intersection and the pupil must now trace it by hand.

The plan of the apex *x* is determined by the projector *x z*, dropped on the parallel A B produced. Draw tangents from *z* to the curve of the ellipse to represent the plans of the sloping edges of the cone, and the plan of the whole cone is complete.

Note.—The lines drawn from *z* to the ellipse will not meet the curve in the ellipse in X and Y.

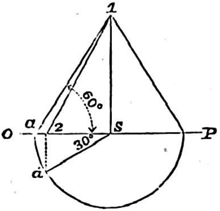
QUESTION.

The vertical height of a cone which inclines at 30° to the H.P. is 12 ft. and its base 8 ft. It stands 4 feet above the H.P. and 10 feet in front of the V.P. Draw first its elevation and then project its plan, the axis being parallel to the V.P. Scale 10' to the inch.

SECTION III.

INCLINATION TO BOTH PLANES.

Lines inclined to one plane like those we have already described in Sect. II., are said to be inclined at a "simple angle." But the pupil will have no difficulty in understanding that a line may be inclined to *both planes* at the same time. Perhaps the best illustration that can be given to show a line in this position, may be found in the lines that form the elevation and plan of a cone.



A cone may be said to be made up of a series of lines drawn from its apex to its circular face, and if so, by referring to Prob. 8, we see that "1 *a*" in the adjoining illustration, is the *elevation* of the line which bounds the most distant view of the left side of the cone, and that "*a s*" is the *plan* of that line.

Now 1 *a* is inclined at an angle of 60° to the H.P., but at the same time it is parallel with the V.P.; in fact, it is in the same plane as the axis "1 *s*." Suppose, however, that we revolve the cone on its axis, and if we do so every point on its circular base must revolve with it. Let us revolve the cone on its axis 30°, then the line 1 *a* revolves with it, and the plan *a* assumes the position indicated by

a'. The line 1 *a* has now moved from its position, which was parallel to the V.P., to a new position in which it is inclined to the V.P., and the pupil must not forget that it is still inclined as it was before at an angle of 60° to the H.P. Hence it has a double inclination, and so it is said to be inclined at a *compound angle*. Let us now examine the effect on 1 *a* in its new position. In the first place *s a'* is the plan of 1 *a* inclined to the V.P. as well as 60° to the H.P. Therefore the point 2 determined by a projector drawn from *a'* is the altered position of *a* in its new elevation, and the line 1 2 represents the elevation of 1 *a* at the compound angle.

I want the pupil now to notice two things in reading this—

1st. The line 1 2 inclined to both planes overhangs exactly the same space in the plan as 1 *a*, which is only inclined to the H.P.; therefore

The plan of a line inclined at an angle to both planes is just of the same length as the plan of a line inclined only to the H.P.

2nd. That the apex 1, the end of the line 1 *a*, has not changed its position in moving at the compound angle, although its other end has; therefore

The height of a line when inclined to both planes is the same as the height of a line inclined only to the H.P. But the elevation of a line inclined to both planes is shorter than an elevation of the same line inclined only to the H.P.

3rd. That what is true of a line, is also true of a surface or a solid, which is only bounded by lines; therefore in projecting lines, surfaces, and solids, inclined to both planes, the pupil must remember that

(*a*) *They cover the same space in the plan as when inolined to the* H.P. *only.*

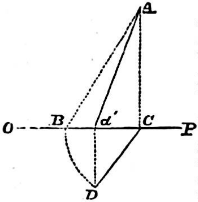
(*b*) *They have the same vertical height in the elevation as when inclined to the* H.P. *only.*

(*c*) *They are fore-shortened in elevation, that is, they occupy less space in breadth than when inolined to the* H.P. *only.*

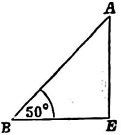
Therefore any object may be projected when inolined to both planes, by finding out the plan and elevation of that object, when inclined to the H.P. only, and then transferring that plan exactly as it is to the angle at which it inclines to the V.P. The position of the various points in the elevation may be determined by drawing projectors from the corresponding points in the plan so obtained to intersect parallel projectors drawn from corresponding points in the elevation of the figure inclined only to the H.P., as illustrated in the Problems which follow.

PROBLEM XXXIII.

To project a line 1" *long, inclined at an angle of* 60° *to the* H.P. *and* 50° *to the* V.P.

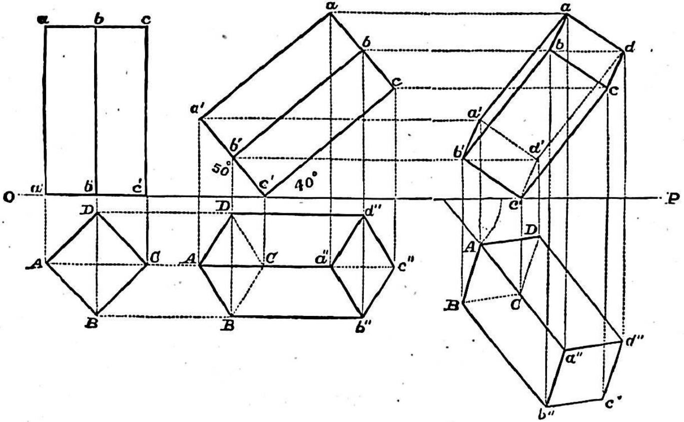


Draw a line of intersection O P. On O P take any point B and draw a line B A, 1' long at an angle of 60° to O P. Then this line is the elevation of a line 1 inch long, inclined at 60° to the H.P. From A let fall a projector A C. Then B C. is the length of the plan of the line inclined at an angle of 60° to the H.P Proceed in the next place to find the length of the elevation of the line when inclined at 50° to V.P.



The construction for this is similar to that for finding the length of the plan. A B is made equal to the real length of the line 1 inch, and the angle A B E 50° the inclination of the line to V P., then B E gives the length of the elevation. Transfer this length B E to the position shown by A d' in the figure so that A d' = B E. From d' drop a projector meeting the plan of the cone in the point D. Join O D. Than the line A d' is the elevation of the line 1" long inclined at an angle of 60° to the H.P, and 50° to the V.P., and C D is its plan.

Fig. 1. Fig. 2. Fig. 3.



PROBLEM XXXIV.

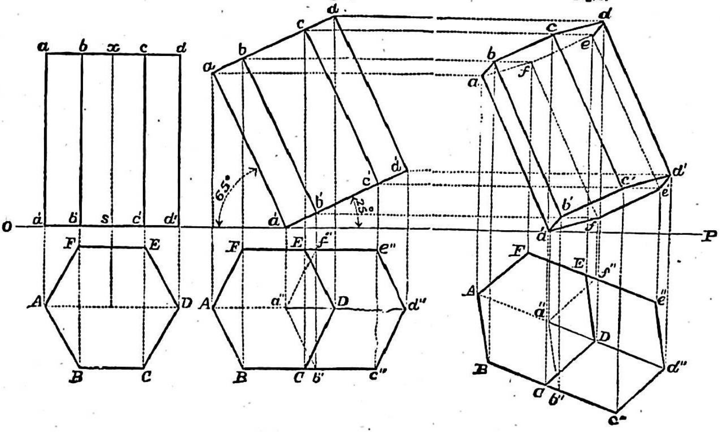
To project a **rectangular prism** *resting on one of its solid angles so that its axis is inclined to the H.P., at an angle of* 40°, *and to the* V.P. *at an angle of* 50°. *The width of its side is* ½ *inch, and its height* 1½ *inch.*

The pupil will find no difficulty in projecting the prism inclined at these angles if he thoroughly understands the principles we have just explained. We will now refer to Problem XXIX., and quote the figures there used to project the same prism when inclined at an angle of 40° to the H.P. Now so long as the inclination of the prism to the H.P. is unchanged, no change whatever will take place in the shape of its plan, however much it may be inclined to the V.P., because it will cover exactly the same space. This the pupil might prove for himself by looking down on a prism inclined at various angles to the V.P. Again, the heights of the various points in the elevation will also be the same. Hence all the pupil has to do is to transfer the plan of Fig. 2, to the plan Fig. 3, keeping the same dimensions exactly, and merely placing it on the plan of the axis. The plan of the axis is found upon the same principle as the plan of the line in Problem XXXIII. He should then draw projectors from each of the points in the plan of Fig. 3 to meet parallel projectors drawn from each of the corresponding points in the elevation of Fig. 2, to determine the new position of the corresponding points in the elevation of the prism in Fig. 3. Join these points of intersection as shown in the illustration, and the elevation of the prism in its doubly-inclined position is complete.

QUESTION.

Project a **triangular prism** 1½ inch high, and having faces ¾ inch in width, when it rests on one of its solid angles with its axis inclined 30° to the V.P. and 45° to the H.P.

Fig. 1. Fig. 2. Fig. 3.



PROBLEM XXXV.

To **project a hexagonal prism** *resting on one of its solid angles; its axis being inclined at an* **angle of 65°** *to the* H.P. *and at an* **angle of 20°** *to the* V.P. *Its height is* 10′ *and the width of each of its faces* 4′ *(Scale ⅛" to the foot.)*

This is simply a repetition of the methods explained in the last problem. We will quote the illustration of the same prism used in Problem 30, where its axis is inclined to the H.P. at an angle of 65° and from the projections found in that illustration Fig. 2, the pupil can easily draw the plan and elevation of the prism in its required position, that is to rest also at an angle of 20° to the V.P. Place the plan Fig 2, with its axis, at an angle of 20° to position of plan found by Problem 33, and from the various points of the plan in Fig. 3 draw projectors to intersect the horizontal projectors drawn from the corresponding points in the elevation of Fig. 2. Join the contiguous points, and the elevation of the prism inclined at 65° to the H.P., and 20° to the V.P. is complete.

QUESTIONS.

1. Project a **cone**, the vertical height of which is 1½ inch, and diameter of base ¾ inch, when its axis inclines at 35° to the V.P. and 45° to the H.P.

2. Project a **pentagonal prism**, the height of which is 2 inches, and the width of its faces 1 inch, when its axis inclines at 65° to the H P. and 30° to the V.P.

3. Project a **square pyramid**, the axis of which is 3 inches, and the width of its faces 1 inch; the axis being inclined at 40° to the H.P. and 50° to the V.P.

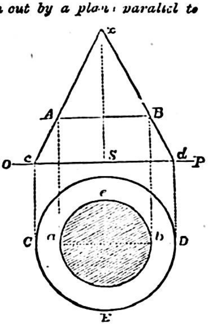
SECTION IV.

ON THE SECTIONS OF SOLIDS.

Hitherto we have explained how objects are projected as seen directly in front or above, and therefore we have merely described the appearance of the outside surfaces of solids. But in order to comprehend the nature and construction of a solid, we require a representation of those parts which cannot be given by either plan or elevation, and is only obtained by means of a view of its interior. This is done by what we call a "*sectional drawing*," that is, a drawing of what is seen when a plane cuts the solid, so as to expose the internal portion of the solid it is desired to show. The following Problems will illustrate how sectional drawings of objects are made.

PROBLEM XXXVI.

To draw the section of a cone *when cut by a plane parallel to its base.*



Let $c\,x\,d$ represent the elevation of a cone and the circle C D E its plan, and let A B be the elevation of a section plane cutting the cone parallel to its base $c\,d$. Draw C D the diameter of the plan parallel to O P, and from A and B let fall projectors cutting C D in a and b. Then the line $a\,b$ is the diameter of the plan of the section. On it construct the circle $a\,e\,b$, which is the plan of the section of the cone cut off by the plane $a\,b$. Shade the section by parallel lines drawn at angles of 45°, which are always used to indicate sectional drawings.

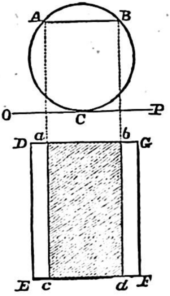
Note.—The section of a cone cut off by an oblique plane is an ellipse, the projection of which belongs to a more advanced stage of Solid Geometry.

PROBLEM XXXVII.

To show the section of a cylinder *lying on the ground with its ends parallel to the* V.P., *and at right angles to the* H.P.; *the plane of section being parallel to the axis of the cylinder.*

Let A B C represent the elevation of a cylinder so placed, and D E F G its plan. Let A B represent the elevation of the section plane cutting the cylinder parallel to its axis.

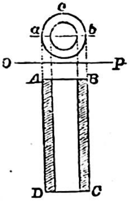
Since the section plane is parallel to the axis, it is quite evident that the plan of the section will be a rectangle. Determine the position of this rectangle on the plan by letting perpendiculars fall from A and B cutting the ends of the plan in *a* and *c*, and *b* and *d*; then, *a c d b* is the plan of the required section. Shade it as in the last Problem.



PROBLEM XXXVIII.

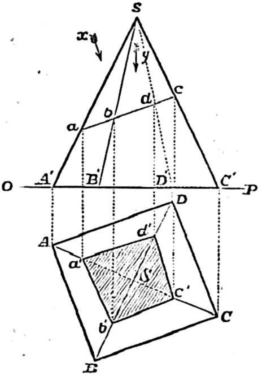
To show the Section of a hollow pipe.

Let *a c b* be the elevation of the end of a pipe of which A B C D is the plan, and let the line *a b* be the elevation of the section plane. Now if the half of the pipe above *a b* were removed, the plan of the section would appear as shown in the illustration. The thickness of the metal is indicated in the plan by diagonal shading lines.



PROBLEM XXXIX.

To show the plan of the **section of a pyramid,** *the sectional lane cutting it obliquely.*



Let A' s C' be the elevation of a pyramid of which A B C D is the plan, and let the line *a b c d* be the elevation of the plane which cuts the prism in an oblique direction. Then to get an oblique plan of the section of the prism, we draw projectors from each of the points *a, b, c, d,* falling on the plans of the corresponding sides. For example: the projector from *a* falls on A S, the plan of the side A' *s* in point *a'*. The projector from *b* falls on B S, the plan of the side B' *s* in the point *b'*, And so on the projector from *c* falls on S C in c', and the projector from *d* falls on S D in point *d'*. Join *a', b', c', d'*, and shade the trapezium *a' b' c' d'* which is an oblique plan of the section of the pyramid cut off by the plane *a b c d*. The pupil however must not fancy that this plan shows the real size of the section, which is seen by an eye looking on the section plane *a b c d* in a vertical direction as indicated by the arrow *x*. The trapezium *a' b' c' d'* is what is called an oblique plan of the section because it is seen obliquely in the direction indicated by the arrow *y*. The method for finding the true size of the section of a pyramid as cut off by an oblique plane, and seen vertically in the direction indicated by the arrow *x*, is shown in Problem 40.

QUESTIONS.

1. Copy the plan and elevation given in Problem XII, and show an oblique section caused by a plane passing through the middle point of the axis of the pentagonal prism at an angle of 40°

2. Show an oblique section caused by a plane passing through the middle point of the axis of the hexagonal prism, Problem XIII, at an angle of 60°.

APPENDIX.

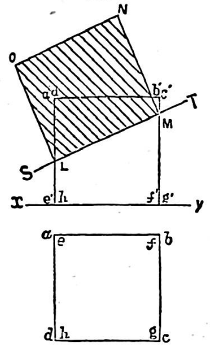
We have now explained all the elementary principles by means of which sections of objects are projected. Young pupils, however, may find some difficulty in applying these under varying conditions; and, as nothing is better than *practice* to familiarise the student with *theory*, we now consider it desirable to introduce them to the following fifty-nine exercises on the sections of solids.

It is all the more important that the pupil, in preparing for the Second Grade Examination in Practical Geometry, should thoroughly master these, because it has been found that the Government Papers require the knowledge; and no student is sure of a pass who is not able to work every one of these exercises.

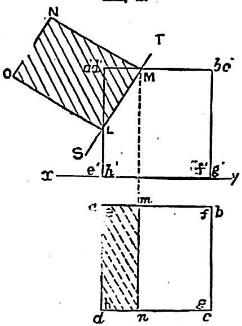
This Appendix contains solutions of exercises with sufficient notes to guide the pupil on—

Sections of Cubes.
Sections of Polygons.
Sections of Irregular, Square, and other Prisms.
Sections of Triangular, Square, Pentagonal, and Hexagonal Pyramids.
Sections of Spheres, Cylinders, and Cones.

Ex. 1.



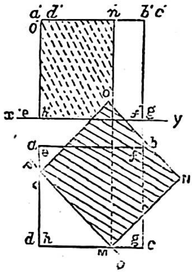
Ex. 2.



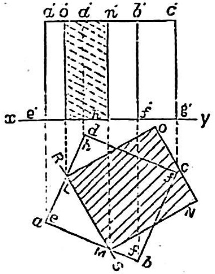
Ex 1. represents a cube A B C....G and S T a plane cutting it. The *plan* of the section is the same as the plan of the cube, viz., the square ($a\ b\ c\ d$). Its real shape is found by drawing from the points L and M the perpendiculars O L and M N respectively, and making them equal to the side of the cube. On joining O N, we obtain the rectangle L M N O, which is the real shape of the section.

Ex. 2.—The plane of section in this example cuts off only a corner of the cube. The real shape of the section is drawn in the same manner as that in Ex. 1. The plan of the part cut is the rectangle ($a\ d\ m\ n$). (*Note*,—O L equals length $a\ d$).

Ex. 3.

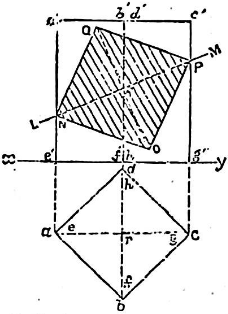


Ex. 4.

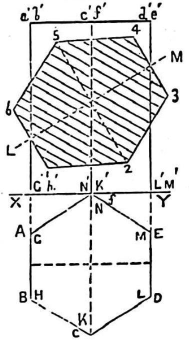


Ex. 3 and 4 represent cubes cut by vertical planes, P Q and R S respectively. The elevations of the sections need no explanation. The real shapes are obtained by making the lines L O and M N equal to the heights of the points O and N, as shewn by their elevations. The constructions would be exactly the same were the solids square base prisms.

Ex. 5.



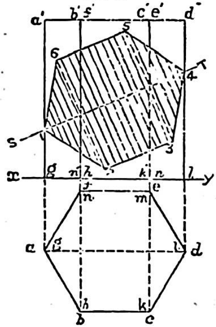
Ex. 6



Ex. 5.—A square prism is given by its plan and elevation, and the line of section L M, to determine the section. It is obtained by drawing a perpendicular line O Q through the point R where L M intersects the axis of the solid and making R O and R Q equal to either (*r d*) or (*r b*) in the plan. Join O N, O P, Q N and Q P, and N O P Q is the shape required.

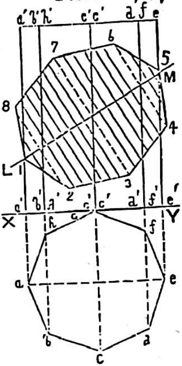
The plan of the section is the square (*a b c d*).

Ex. 7.

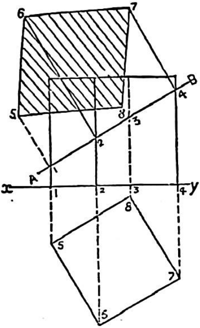


Ex. 6, 7, and 8, are worked upon exactly the same principle, viz that of obtaining the widths of the sections, and marking half of each on both sides of the line of section as a centre line.

EX. 8

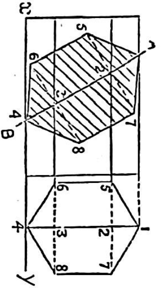


Ex. 9.



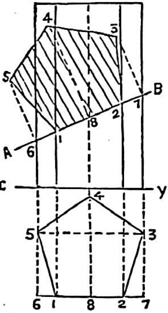
Ex. 9.— ection of a square prism. To draw this section the method explained in Ex. 5 is not so easy of application as that adopted in the sketch. The section is found by drawing lines from the points of intersection of the line A B. and edges of the solid perpendicular to the section line, and equal to the distances of the corresponding edges from the ground line X Y.

Ex. 10.



Ex. 10.—Given a hexagon prism and its end view to determine the section formed by the plane A B. The construction for obtaining this section is similar to that of Ex. 7, the end view being used as a plan.

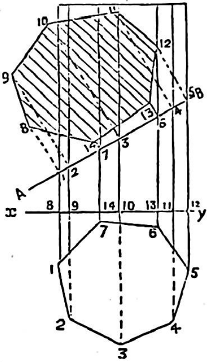
Ex. 11.



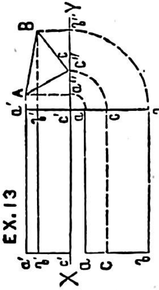
Ex. 11.—Pentagonal Prism and section on line A B. The distance 3, 7, &c, in the section, correspond to those similarly figured in the plan.

Ex. 12.—A section is here shewn of a heptagon prism. The shape is found in the same manner as that in Ex 9

Ex. 12.

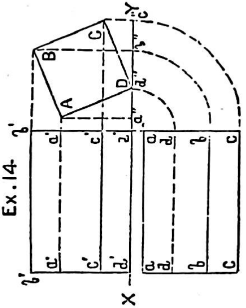


EX. 13



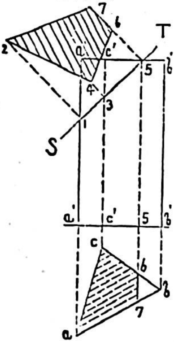
Ex 13. We have here given the elevation and end view of a triangular prism, and are required to draw its plan. Project the points A B and C on to the ground line X Y to $a''b''$ and c''. With any convenient point as a centre (in this case the corner (c'') of the elevation is used) draw the arcs $a''a$, $b''b$, and $c''c$. Draw $a\ c\ b$ at right angles to X Y perpendicularly under the same points in elevation, and we obtain the required plan.

Ex. 14



Ex. 14.—The same construction is here gone through to find the plan of a square prism given in a similar manner to Ex. 13.

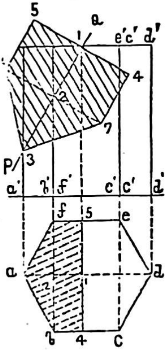
Ex. 15



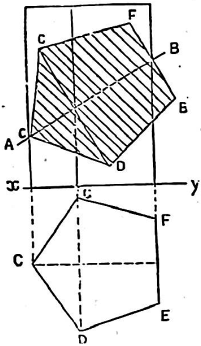
Ex. 15.—Shows a section of a triangular prism, the plane only cutting off a corner of the solid. The plan of the section ($ac67$) is first obtained, and the real shape as in Ex. 9.

Ex. 16.—A Hexagonal prism similarly cut to Ex. 15, and the section found as in Exs. 6 and 7.

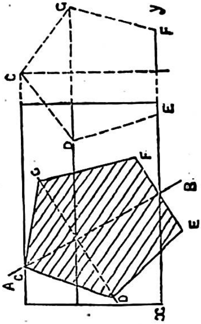
Ex. 16



Ex. 17.

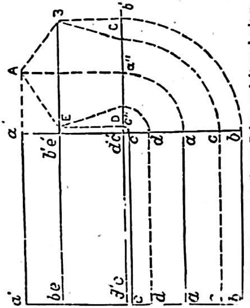


Ex. 18.



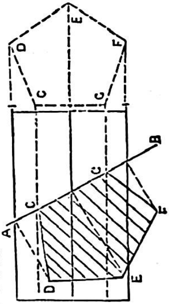
Exs. 17 and 18.—Sections of Pentagon Prisms. Ex. 17 is worked upon the same principle as Ex. 6, and Ex. 18 same as Ex. 10.

Ex. 19.



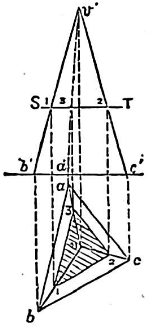
Ex. 19.—Given a Pentagon Prism and its end view to find its plan. This needs no explanation after Exs. 13 a d 14.

Ex. 20

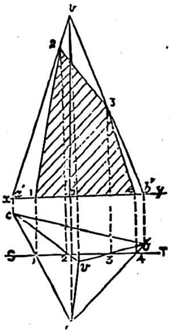


Ex. 20.—Another section of a Pentagon Prism. Prism is given by its plan and end view. (See Exs. 10 & 11.)

Ex. 21.



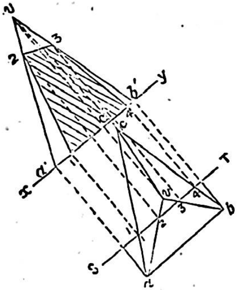
Ex. 22.



Ex. 21.—Section of a triangular pyramid by a horizontal plane S T. Find the plans of the points 1, 2 and 3, in which the line S T cuts the edges of the pyramid. On joining these points the triangle formed is both plan and real shape of the section.

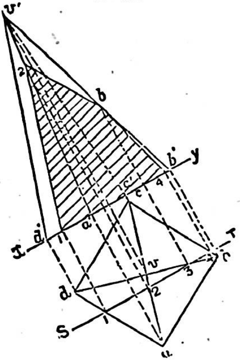
Ex. 22.—In this case the pyramid is cut by a vertical plane: 1 and 4 are the points in which it cuts the base, and 2 and 3 the points in which the plane cuts the edges *v c* and *v b* respectively. On joining the elevations of these points 1, 2, 3 and 4, a four-sided figure is obtained, which is the required section (elevation and real shape).

Ex. 23.

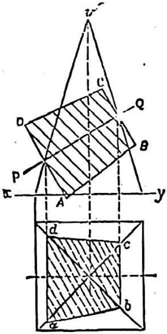


In Exs. 23 and 24 the plans and heights of the pyramids are supposed to be given. For convenience sake the ground line is drawn parallel to the line of section and we obtain both an elevation and the real shape of the section.

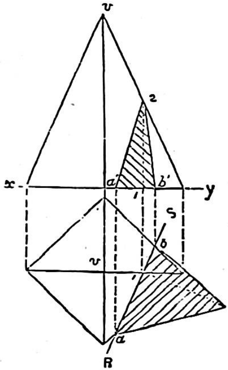
Ex. 24.



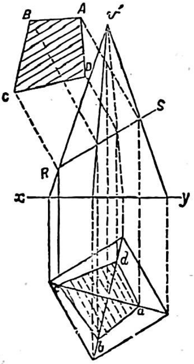
Ex. 25.



Ex. 26.



Ex. 27.

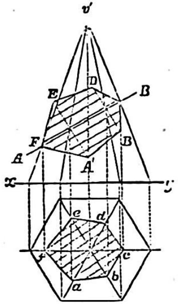


The Examples on this page represent sections of square pyramids.

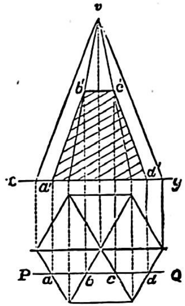
In Ex. 26 a separate construction has to be gone through to find the real shape of the section, the elevation being narrower than the real shape, the line 1, 2, on the plan, is made equal to the height of the point 2. a' b' 2 is the elevation, and a b 2 the real shape.

In Exs. 25 & 27 the plan of the section has to be found first, and then the distances for the real shape obtained from this plan.

Ex. 29.

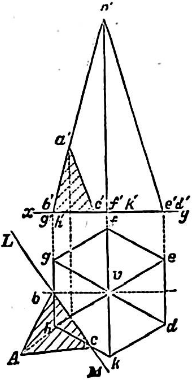


Ex. 30.

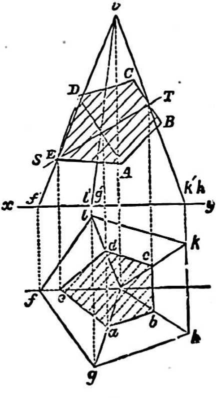


Ex. 29, 30, and 31, show three different sections of hexagonal pyramids.
Ex. 29 is worked like Ex. 25. Ex 30 like Ex. 22, and Ex. 31 like Ex. 26.
Ex. 32 is a section of a Pentagon Pyramid worked out in a similar manner to Ex. 29.

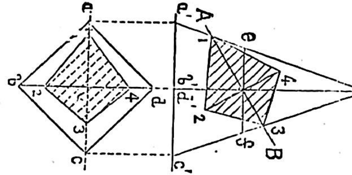
Ex. 31.



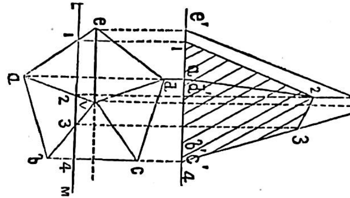
Ex. 32.



Ex.35



Ex.33

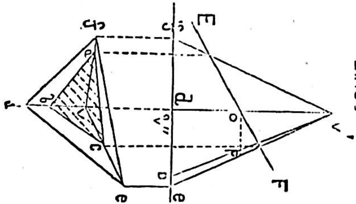


The first two figures on this page need no explanation, being merely repetitions of previous examples.

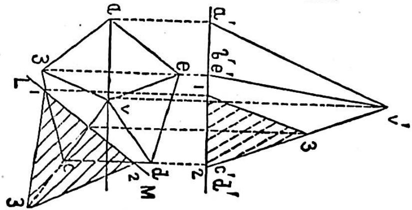
In Ex. 35, to obtain the width of the section at the point where the line *A B* cuts the axis, we have to draw a line *e f* across the figure. This line is the width of the solid across that particular part, so that the line 2 4 in both plan and elevation are made equal to *e f*.

In Ex. 36, however, the point *b* in the section has to be found by making another view of the line *v d*. *v′ D* in the elevation is made equal to *v d*, when *v′ D* will represent the end view of the edge of the solid *V D*. If from *O* we draw a horizontal line intersecting *v′ D* in *P*, *O P* is the horizontal distance of the point *o* from the edge *V D*, and consequently the length *v b* required in the plan. Only plan of section is here shewn, the real shape is to be drawn as in Ex. 27.

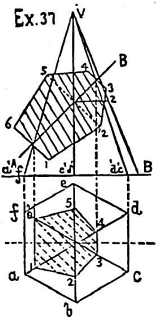
Ex.36



Ex.34

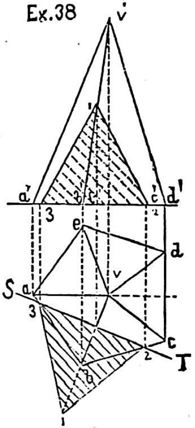


Ex. 37



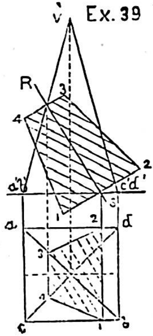
In Ex. 37, the the points 2 and 5 have to be found in a similar manner to the point in Ex. 36.

Ex. 38

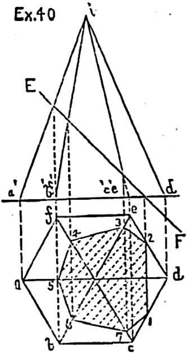


Ex. 38, 39, and 40, need no explanation.

Ex. 39

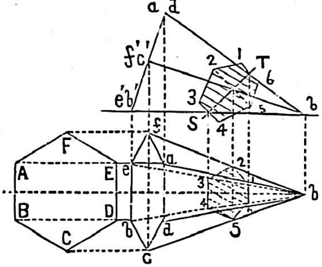


Ex. 40



The pupil can draw the real shape of the section in Ex. 40 for himself.

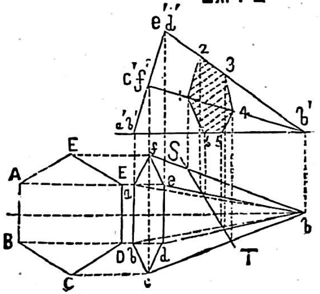
Ex. 41



Two examples are here given of hexagon prisms lying on the horizontal plane.

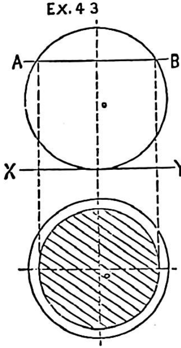
The plan and real shape of the section of No. 41 by a plane (S T) is shown. The widths for the real shape are, as usual, obtained from the plan.

Ex. 4 2

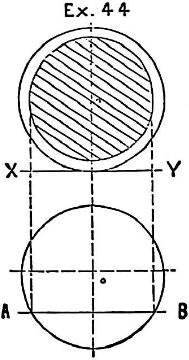


In Ex. 42 only the elevation of the section is drawn to avoid complication of the figure. The pupil will draw the real shape upon the same construction as Ex. 20.

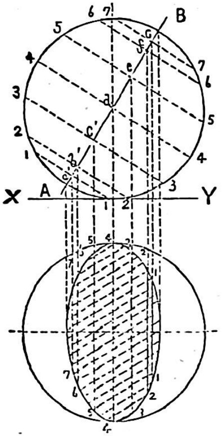
Ex. 43.—Sphere cut by a horizontal plane A B. The shaded circle gives the section.



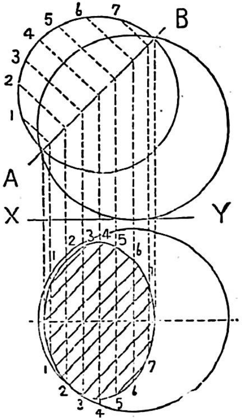
Ex. 44.—A sphere cut by a vertical plane A B. Section shown by the shaded circle.



Ex.45



Ex. 46



Ex. 45.—A sphere cut by an oblique plane A B, which passes through the centre of the solid. The portion of the line within the circle is marked off into a number of divisions which need not be equal. 1-1, 2-2, &c., drawn at right angles to A B, give the widths of the section at the different points *a*, *b*, &c.

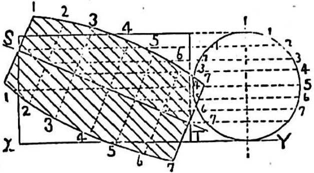
These widths marked on the plans of the lines give a number of points through which a curve can be drawn giving the plan of the section.

In Ex. 46 the plane A B does not pass through the centre, and the widths are found by drawing a circle on the portion of the line intercepted by the circumference of the circle. Only half of the circle is shown in the sketch, to simplify the figure.

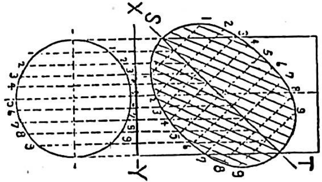
Ex. 47.—Section of a cylinder by a plane S T. The widths of the section are here obtained at nine different points. Corresponding lines drawn at right angles to the line S T, and equal to their respective widths, give the points through which the curve may be drawn, giving the real shape of the section. 48, 49 and 50 are worked upon the same principle.

In 50 the elevation only is given, and a plan is drawn in the most convenient position.

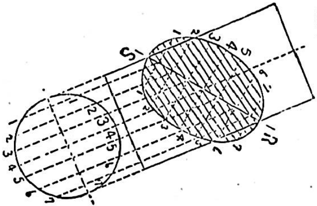
Ex. 49



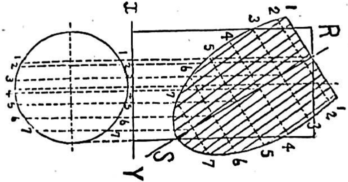
Ex. 47



Ex. 50

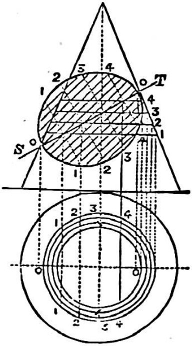


Ex. 48

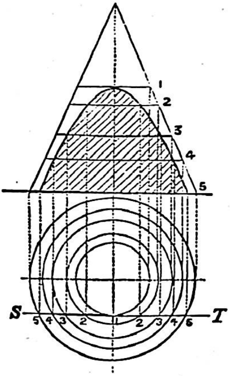


Ex. 52.

Ex. 52.—A cone cut by an oblique plain S T. The portion the line intercepted between the lines composing the elevation of the cone is marked off into a number of divisions 1 2 3 4 &c. To obtain the width of No. 1, a horizontal section has to be drawn through that point; the plan of this is a circle, the point 1 being projected on to the plan, the portion (1.1.) intercepted between the circle gives the width at that particular point. As many horizontal sections have to be obtained as points are marked off on the line S T. The rest of the construction is like Ex. 47. The plan of the section is a curve drawn through the points 0 2 3 4 4 3 2 1 0 in the plan.

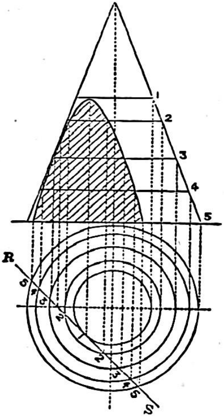


Ex. 53.



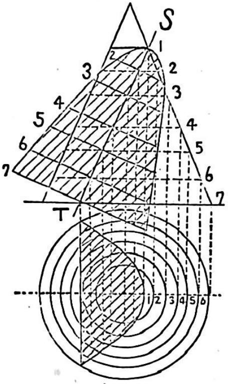
Ex. 53.—Section of cone by plane S T. As in Ex. 53 the cone is divided into a number of horizontal sections 1 2 3 4 & 5. The points where the circles cut the line S T, give points in the section which when projected to their respective elevations give points through which the curve can be drawn.

Ex. 54.

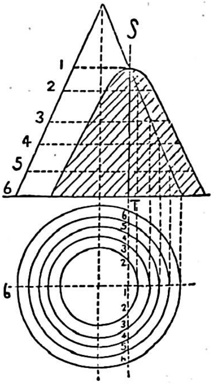


Ex. 54 is worked in a similar manner. The elevation only is shewn. Real shape is found as in Ex. 26 and 34.

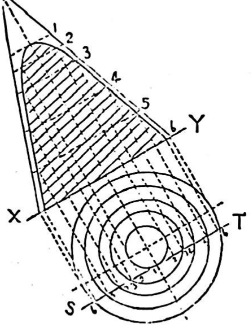
Ex.55



Ex.56



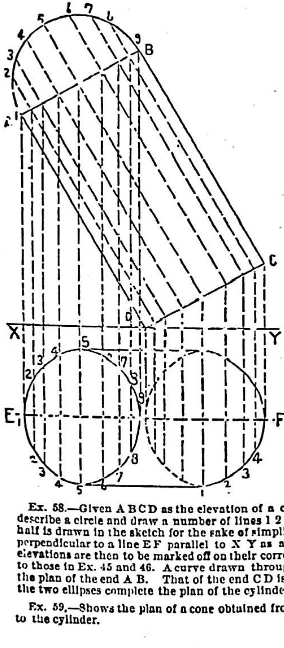
Ex 57



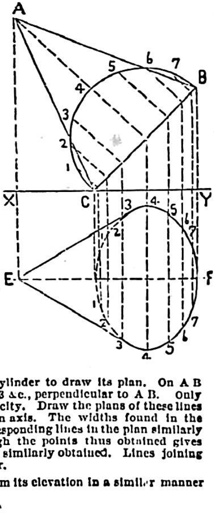
Exs. 55 and 56 are sections of cones worked like that in Ex. 52.

Ex. 57 is similar to 53. The cone here is given by its plan and height, and the ground line is drawn parallel to S T as being the most convenient position.

Ex. 58



Ex. 59



Ex. 58.—Given A B C D as the elevation of a cylinder to draw its plan. On A B describe a circle and draw a number of lines 1 2 3 &c., perpendicular to A B. Only half is drawn in the sketch for the sake of simplicity. Draw the plans of these lines perpendicular to a line E F parallel to X Y as an axis. The widths found in the elevations are then to be marked off on their corresponding lines in the plan similarly to those in Ex. 45 and 46. A curve drawn through the points thus obtained gives the plan of the end A B. That of the end C D is similarly obtained. Lines joining the two ellipses complete the plan of the cylinder.

Ex. 59.—Shows the plan of a cone obtained from its elevation in a similar manner to the cylinder.

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